# Macro II

Professor Griffy

UAlbany

Spring 2025

# Introduction

- Midterm in two weeks!
- Homework 3 due tonight!
- Homework 4 due 3/25.
- Today: Real Business Cycle Model
- Original paper: Kydland and Prescott (1982)

# Basic RBC Model

- Household solves

$$\max_{\{C_t, I_t, L_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t N_t \left[ \ln\left(\frac{C_t}{N_t}\right) + \chi \frac{(1 - L_t/N_t)^{1-\gamma} - 1}{1-\gamma} \right]$$

$$\begin{aligned} s.t. \quad & C_t + I_t = r_t K_t + W_t L_t, && \text{(BC)} \\ & K_{t+1} = (1-\delta) K_t + I_t, && \text{(CA)} \\ & L_t \in [0, N_t], \\ & K_0 \text{ given}, \quad C_t \geq 0. \end{aligned}$$

- Parameter restrictions: $\chi > 0$, $\gamma \geq 0$, $0 < \beta < 1$
- $1 - L_t/N_t$ is per capita leisure
- Note that $K_t < 0$ represents borrowing

# Basic RBC Model II

- Assume constant growth in population and productivity

$$
\begin{aligned}
N_t &= N_0 N^t, \quad N_0, N > 0, \quad \beta N < 1, \\
A_t &= A_0 A^t, \quad A_0, A > 0.
\end{aligned}
$$

- The per-effective-worker problem becomes:

$$
\max_{\{c_t, k_{t+1}, \ell_t\}_{t=0}^{\infty}} \sum\nolimits_{t=0}^{\infty} (\beta N)^t \left[ \ln (A_t c_t) + \chi \frac{(1-\ell_t)^{1-\gamma} - 1}{1-\gamma} \right],
$$

$$
\begin{aligned}
s.t. \quad & c_t + ANk_{t+1} = R_t k_t + w_t \ell_t, \\
& \ell_t \in [0,1]; \quad k_0 \text{ given}, \quad c_t \geq 0, \\
& \lim_{J \to \infty} \left( \prod_{j=1}^{J-1} R_{t+j}^{-1} \right) A_{t+J} N_{t+J} k_{t+J} = 0.
\end{aligned}
$$

# Solution

- The first order conditions are

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}, \tag{EE}$$

$$u'(A_t c_t) A_t w_t = v'\left(1 - \ell_t\right)$$

$$\Leftrightarrow \frac{1}{c_t} w_t = \chi \left(1 - \ell_t\right)^{-\gamma}. \tag{LL}$$

- Euler equation and “portfolio allocation”

# Effect of interest rate changes on savings

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}$$

- Substitution effect: Increasing $R_{t+1}$ lowers the price of future consumption, inducing substitution into the cheaper good (future consumption), inducing more saving
- Income effect
  - Positive assets: Increasing $R_{t+1}$ raises future income and consumption, lowers future $MU_C$, inducing less savings
  - Negative assets: Increasing $R_{t+1}$ reduces future income and consumption, raises future $MU_C$, inducing more savings

# Effect of interest rate changes on savings

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}$$

- General (empirical) consensus
  - Consumers are net savers: the aggregate income effect of higher interest rates is to lower saving
  - The substitution effect weakly dominates implying that savings increases in interest rates

# Labor-leisure tradeoff

$$\frac{1}{c_t} w_t = \chi \left(1 - \ell_t\right)^{-\gamma}$$

- $MU_C \times wage = MU_L$
- Wealth effects: Holding $w_t$ constant, higher permanent income raises current consumption, lowers marginal benefit of working
  - Higher assets
  - Higher current or future non-labor income
  - Higher current or future labor income
  - Increasing non-labor component of permanent income lowers labor supply

# Effects of increasing the current wage ($MU_C \times wage = MU_L$)

$$\frac{1}{c_t} w_t = \chi \left(1 - \ell_t\right)^{-\gamma}$$

- Substitution effect: holding $MU_C$ constant, and raising $w_t$ increases marginal benefit of working
- Income effect: raising $w_t$ increases $y_t^P$, lowers $MU_C$ and marginal benefit of working

# General (empirical) consensus

$$\frac{1}{c_t} w_t = \chi \left(1 - \ell_t\right)^{-\gamma}$$

- Temporary wage increases generate more hours due to small income effect
- Permanent wage increases generate no more hours because income and substitution effects offset. Consistent with long-term data where wage rises but labor hours do not
- Our specification delivers this

# Labor supply curve

- Rearrange (LL) to get

$$\ell_t = 1 - (c_t \chi)^{1/\gamma} w_t^{-1/\gamma}.$$

- Frisch supply curve

$$\ell_t = f(w_t, MU_C) = f\left(w_t, y_t^P\right).$$

  - Consider effects of changing wages with $MU_C$ held constant
  - Wealth effects ignored
  - Note: $MU_C$ can depend on things besides $y_t^P$, although it does not here

# Intertemporal elasticity of substitution of labor ($IES_L$ or Frisch elasticity)

- Measures willingness to vary labor over time, holding $MU_C$ (wealth) constant

$$IES_L = \left.\frac{d \ln \left(\ell_1/\ell_2\right)}{d \ln \left(w_1/w_2\right)}\right|_{MU_C}.$$

# Derivation

$$\frac{1}{c_t} = \beta A^{-1} \frac{1}{c_{t+1}} R_{t+1}, \tag{EE}$$

$$\frac{1}{c_t} w_t = \chi \left(1 - \ell_t\right)^{-\gamma}. \tag{LL}$$

- Combine (EE) and (LL)

$$\chi \frac{\left(1-\ell_1\right)^{-\gamma}}{w_1} = \beta A^{-1} \chi \frac{\left(1-\ell_2\right)^{-\gamma}}{w_2} R_2.$$

# Portfolio Allocation

- Note that the household smooths leisure as well as consumption
- For example, interest rates affect labor supply
- Rearrange the previous equation

$$
\begin{aligned}
\beta A^{-1} R_2 \left( \frac{w_1}{w_2} \right) &= \frac{(1-\ell_1)^{-\gamma}}{(1-\ell_2)^{-\gamma}}, \\
\ln \left( \beta A^{-1} R_2 \right) + \ln \left( \frac{w_1}{w_2} \right) &= -\gamma \ln (1-\ell_1) + \gamma \ln (1-\ell_2), \\
&= -\gamma \big[ \ln (1 - \exp(\ln \ell_1)) \\
&\qquad - \ln (1 - \exp(\ln \ell_2)) \big].
\end{aligned}
$$

- Implicitly differentiate:

$$d\ln\left(\frac{w_1}{w_2}\right) = \gamma\frac{\exp(\ln(\ell_1))}{1-\exp(\ln(\ell_1))}d\ln(\ell_1) - \gamma\frac{\exp(\ln(\ell_2))}{1-\exp(\ln(\ell_2))}d\ln(\ell_2).$$

- Now assume that $\ell_1 = \ell_2 = \ell$

$$\begin{aligned} d\ln\left(\frac{w_1}{w_2}\right) &= \gamma\frac{\ell}{1-\ell}d\ln(\ell_1) - \gamma\frac{\ell}{1-\ell}d\ln(\ell_2) \\ &= \gamma\frac{\ell}{1-\ell}\left[d\ln(\ell_1) - d\ln(\ell_2)\right] \\ &= \gamma\frac{\ell}{1-\ell}d\ln\left(\frac{\ell_1}{\ell_2}\right). \end{aligned}$$

- Finally, we get

$$IES_L = \left. \frac{d \ln(\ell_1/\ell_2)}{d \ln(w_1/w_2)} \right|_{MU_C} = \frac{1}{\gamma}\left(\frac{1-\ell}{\ell}\right).$$

- Tip: if $\gamma = 0$ such that utility is linear in leisure, then $IES_L$ is infinite

# Non-Separable Preferences (Low, 2005)

- Household solves

$$\max_{\{c_t,k_{t+1},\ell_t\}_{t=0}^{\infty}} E_0\left(\sum_{t=0}^{\infty}(\beta N)^t u(A_t c_t, 1-\ell_t)\right),$$
$$s.t. \quad c_t + ANk_{t+1} = R_t k_t + w_t \ell_t,$$
$$\ell_t \in [0,1],$$

- and the other usual constraints
- The first-order conditions are

$$\begin{aligned} u_{Ac}(A_t c_t, 1-\ell_t)A_t &= \lambda_t, \\ u_{1-\ell}(A_t c_t, 1-\ell_t) &= \lambda_t w_t, \\ \lambda_t &= \beta A^{-1} E_t(R_{t+1}\lambda_{t+1}). \end{aligned}$$

where $\lambda_t$ is the multiplier on the budget constraint

- Benchmark utility specification is isoelastic Cobb-Douglas

$$u\left(A_t c_t, 1-\ell_t\right) = \frac{1}{1-\gamma}\left((A_t c_t)^{\chi}(1-\ell_t)^{1-\chi}\right)^{1-\gamma}$$

- The derivatives of this function are

$$\begin{aligned} u_{Ac} &= \chi(1-\gamma)\frac{1}{A_t c_t} u\left(A_t c_t, 1-\ell_t\right), \\ u_{1-\ell} &= (1-\chi)(1-\gamma)\frac{1}{1-\ell_t} u\left(A_t c_t, 1-\ell_t\right), \\ u_{1-\ell,Ac} &= \frac{\chi(1-\chi)(1-\gamma)^2}{(1-\ell_t)A_t c_t} u\left(A_t c_t, 1-\ell_t\right). \end{aligned}$$

- Key issue: Is consumption at time-$t$ a substitute or a complement for leisure at time-$t$?
  - This depends on the sign of the cross-partial derivative $u_{Ac,1-\ell}(\cdot)$: $u_{Ac,1-\ell} > 0$ implies complements
  - For the benchmark specification

$$u_{1-\ell,Ac} = \chi(1-\chi)(1-\gamma) \times (A_t c_t)^{\chi(1-\gamma)-1}(1-\ell_t)^{(1-\chi)(1-\gamma)-1}.$$

  - This term will be negative if $\gamma > 1$
  - Baseline assumption: $\gamma = 2.2$, implying consumption and leisure are substitutes

- Combining first-order conditions yields

$$u_{1-\ell}\left(A_t c_t, 1-\ell_t\right) = u_{Ac}\left(A_t c_t, 1-\ell_t\right) A_t w_t.$$

- With the baseline preferences, this becomes

$$\begin{aligned} \frac{1-\chi}{1-\ell_t} &= \chi \frac{w_t}{c_t}, \\ \Rightarrow \ell_t &= 1 - \left(\frac{1-\chi}{\chi}\right) \frac{c_t}{w_t}. \end{aligned}$$

- This specification produces constant hours along a balanced growth path
- King et al (1989) provide a general set of conditions

# Data Puzzle 1

- Consumption tracks income over the life-cycle: Inconsistent with consumption smoothing
- If consumption and leisure are substitutes, people working more hours will consume more implying that consumption tracks income (Heckman, 1974)

# Data Puzzle 2

- There is a discrete drop in consumption immediately after retirement which is inconsistent with consumption smoothing
- If consumption and leisure are substitutes, then consumption will drop at retirement (French 2005, Aguiar and Hurst 2005)

# Data Puzzle 3

- Low-wage young people work many hours; high-wage old people work fewer hours: Inconsistent with the intertemporal substitution of labor
- Young people work long hours to fund precautionary saving
- This precautionary saving builds up assets and reduces the need to work when old
- This result does not require non-separable preferences
- It does require life-cycle (not infinite-horizon) framework with low initial wealth

# Conclusion

- Spring break!!!!! Woooooo!
- Midterm in two weeks!
- Homework 3 due tonight
- Homework 4 due after break (3/25)