

Macro II

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Introduction

- ▶ Today: Asset pricing
- ▶ The “Lucas Tree Model.”
- ▶ Homework due tonight.
- ▶ No class next Tuesday (2/25). Instead, work on HW3.
- ▶ Test Thursday after Spring Break (3/27)



Feb. 25th and 27th

- ▶ I will be in Japan. Time difference means class would be 11:30pm to 1:00am for me.
- ▶ Will post the slides for lecture 11 (stochastic neoclassical growth) and teach lecture 10 (market structure) on 3/4.
- ▶ Please review the lecture 11 slides, and focus on stochastic component.

Rational Expectations Competitive Equilibrium

Definition: Given the set of exogenous stochastic processes $\{x_t\}$, and initial conditions a_0 , a rational expectations equilibrium is a set of stochastic processes for prices $\{p_t\}$ and quantities $\{q_t\}$ such that:

- ▶ Given $\{p_t\}$, $\{q_t\}$ is consistent with optimal behavior on the part of consumers, producers and (if relevant) the government.
- ▶ Given $\{p_t\}$, $\{q_t\}$ satisfies the government's budget constraints and borrowing restrictions.
- ▶ $\{p_t\}$ satisfies any market-clearing conditions.

Lucas Tree Overview

- ▶ We are given equilibrium quantities and equilibrium demand functions—back out equilibrium prices.

- ▶ We know the function $D(p)$ and the quantity q_0 : now find p_0 .

Compare with Literature on Consumption

- ▶ Consumption: Take rates of return as given, solve for consumption.
- ▶ Asset Pricing: Take consumption as given, solve for rates of return.

Model Structure

- ▶ Preferences: n identical consumers, maximizing

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \right),$$
$$\beta \in (0, 1), \quad u'(\cdot) > 0, \quad u''(\cdot) \leq 0.$$

- ▶ Endowment: one durable “tree” per individual. Each period, the tree yields some “fruit” ($d_t \equiv$ dividends).
- ▶ Technology: Fruit cannot be stored. Dividends are exogenous and follow a time-invariant Markov process

$$\Pr(d_{t+1} \leq y | d_t = x) = F(y, x), \forall t,$$

with density $f(y, x)$.

Solution strategy

- ▶ Find the competitive equilibrium allocation via the social planner's problem. Assume equal weights on each person's utility (parallel to equal endowments).
- ▶ i.e., use welfare theorems.
- ▶ Calculate the FOC for individuals with the opportunity to buy/sell specific assets.
- ▶ Evaluate FOC at the competitive equilibrium allocation.

Step 1: Social planner's problem

- ▶ Use a representative agent

- ▶ Social planner solves

$$\begin{aligned} \max_{\{c_t\}_{t=0}^{\infty}} E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \right) \\ \text{s.t. } c_t \leq d_t. \end{aligned}$$

- ▶ Solution: $c_t = d_t, \forall t$ (*non-storable good!*).

- ▶ What does this mean?

Definitions

c_t = consumption,

p_t = price of a tree = price of stock,

x_t = total resources

s_{t+1} = number of trees/shares of stock,

R_t = gross return on one-period risk-free bond,

R_t^{-1} = price of a one-period, risk-free discount bond,

b_{t+1} = risk-free discount bonds.

Step 2: Representative consumer's problem

$$\begin{aligned} \max_{\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}} & E \left(\sum_{t=0}^{\infty} \beta^t u(c_t) \mid I_0 \right) \\ \text{s.t.} & c_t + p_t s_{t+1} + R_t^{-1} b_{t+1} = x_t, \\ & x_t = (p_t + d_t) s_t + b_t, \\ & \lim_{J \rightarrow \infty} \beta^J E_t (u'(c_{t+J}) p_{t+J} s_{t+J+1}) = 0 \\ & \lim_{J \rightarrow \infty} \beta^J E_t (u'(c_{t+J}) b_{t+J+1}) = 0, \\ & s_0, b_0 \text{ given} \end{aligned}$$

- ▶ where I_0 is the information set at time 0.

Consumer's problem

- ▶ Consumer i picks c_t^i , b_{t+1}^i and s_{t+1}^i on the basis of

$$I_t^i = \left\{ \begin{array}{l} \{d_{t-m}, p_{t-m}, R_{t-m}\}_{m=0}^t, \\ \{s_{t+1-m}^j, b_{t+1-m}^j\}_{m=0}^{t+1}, \forall j \neq i, \\ \{c_{t-m}^j, x_{t-m}^j\}_{m=0}^t, \forall j \neq i, \\ \{s_{t-m}^i, b_{t-m}^i, x_{t-m}^i\}_{m=0}^t, \{c_{t-m}^i\}_{m=1}^t, \end{array} \right\},$$

- ▶ It turns out that d_t summarizes the state of the aggregate economy, with $p_t = p(d_t)$ and $R_t = R(d_t)$.
- ▶ It is the only stochastic variable, and aggregate resources equal d_t .
- ▶ Because d_t is a time-invariant Markov process, the consumer's problem is time-invariant

Recursive formulation

Bellman's functional equation:

$$\begin{aligned} V(x_t, d_t) = & \min_{\lambda_t \geq 0, c_t \geq 0} \max_{s_{t+1}, b_{t+1}} u(c_t) + \lambda_t (x_t - c_t - p_t s_{t+1} - R_t^{-1} b_{t+1}) \\ & + \beta \int V((p(d_{t+1}) + d_{t+1}) s_{t+1} + b_{t+1}, d_{t+1}) dF(d_{t+1}, d_t). \end{aligned}$$

► The FOC for an interior solution are:

$$\begin{aligned} u'(c_t) &= \lambda_t, \\ \lambda_t p_t &= \beta \int \frac{\partial V[t+1]}{\partial x_{t+1}} (p(d_{t+1}) + d_{t+1}) dF(d_{t+1}, d_t), \\ \lambda_t R_t^{-1} &= \beta \int \frac{\partial V[t+1]}{\partial x_{t+1}} dF(d_{t+1}, d_t). \end{aligned}$$

Euler Equations

- Note that (by Benveniste-Scheinkman)

$$\frac{\partial V[t]}{\partial x_t} = \lambda_t,$$

so that

$$\begin{aligned} p_t &= \beta E_t \left(\frac{u'(c_{t+1})}{u'(c_t)} (p_{t+1} + d_{t+1}) \right), \\ R_t^{-1} &= \beta E_t \left(\frac{u'(c_{t+1})}{u'(c_t)} \right). \end{aligned}$$

Step 3: Equilibrium

- ▶ Intuition
- ▶ Agents allocate resources based on beliefs about future prices and consumption
- ▶ These decision rules determine processes for market clearing prices and quantities.
- ▶ In a rational expectations equilibrium, the actual processes must be consistent with the beliefs.

- ▶ **Sequential** definition: Given the stochastic process $\{d_t\}_{t=0}^{\infty}$ and the initial endowments $s_0 = 1$ and $b_0 = 0$, a rational expectations equilibrium consists of the stochastic processes $\{c_t, s_{t+1}, b_{t+1}, p_t, R_t\}_{t=0}^{\infty}$ such that:
 - ▶ Given the process for prices $\{p_t, R_t\}$, $\{c_t, s_{t+1}, b_{t+1}\}$ solves the consumer's problem.
 - ▶ All markets clear: $c_t = d_t$, $s_{t+1} = 1$, and $b_{t+1} = 0$, $\forall t$.

- ▶ **Recursive definition:** given the random variable d_0 , the conditional distribution $F(d_{t+1}, d_t)$, and the initial endowments $s_0 = 1$ and $b_0 = 0$, a recursive rational expectations equilibrium consists of pricing functions $p(d)$ and $R(d)$, a value function $V(x, d)$, and decision functions $c(x, d)$, $s(x, d)$, and $b(x, d)$ such that:
 - ▶ Given the pricing functions $p(d)$ and $R(d)$, the value and policy functions $V(x, d)$, $c(x, d)$, $s(x, d)$, and $b(x, d)$ solve the consumer's problem.
 - ▶ Markets clear: for $x = p(d) + d$, $c(x, d) = d$, $s(x, d) = 1$, and $b(x, d) = 0$.

Backing out prices

- ▶ Find $R(d_t)$: impose the equilibrium allocation, $c_t = d_t$, to get

$$\begin{aligned} R_t^{-1} &= \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} \right) \\ &= \beta \frac{1}{u'(d_t)} E_t (u'(d_{t+1})). \end{aligned} \quad (\text{EE})$$

- ▶ Find $p(d_t)$: impose the equilibrium allocation, $c_t = d_t$, to get

$$p_t = \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right). \quad (\text{EE}')$$

Bond Price

- ▶ Recall equation (EE):

$$R_t^{-1} = \beta \frac{1}{u'(d_t)} E_t (u'(d_{t+1})), \quad (\text{EE})$$
$$R_t = \frac{u'(d_t)}{\beta E_t (u'(d_{t+1}))}.$$

- ▶ The price of a discount bond increases (return falls) in β .
- ▶ The price increases (return falls) in expected future marginal utility, and decreases in current marginal utility: reflects consumption smoothing motive.
- ▶ Recall from last time: implies that more uncertainty raises bond price if convex preferences.

Stock prices

- ▶ Recall equation (EE'):

$$p_t = \beta E_t \left(\frac{u'(d_{t+1})}{u'(d_t)} (p_{t+1} + d_{t+1}) \right). \quad (\text{EE}')$$

- ▶ Define the expected rate of return on stocks as

$$E_t (R_t^s) = E_t \left(\frac{p_{t+1} + d_{t+1}}{p_t} \right).$$

Equity premium

- ▶ The expected rate of return on stocks is

$$E_t(R_t^s) = E_t\left(\frac{p_{t+1} + d_{t+1}}{p_t}\right).$$

- ▶ Recall that

$$E_t(XY) = E_t(X)E_t(Y) + C_t(X, Y).$$

- ▶ Rewrite (EE'):

$$\begin{aligned} 1 &= \beta E_t\left(\frac{u'(d_{t+1})}{u'(d_t)}\left(\frac{p_{t+1} + d_{t+1}}{p_t}\right)\right) \\ &= \beta E_t\left(\frac{u'(d_{t+1})}{u'(d_t)}R_t^s\right) \\ &= \beta E_t\left(\frac{u'(d_{t+1})}{u'(d_t)}\right)E_t(R_t^s) + C_t\left(\beta\frac{u'(d_{t+1})}{u'(d_t)}, R_t^s\right) \end{aligned}$$

Risk premium

- ▶ Insert (EE) and rearrange:

$$1 = R_t^{-1} E_t(R_t^s) + C_t \left(\beta \frac{u'(d_{t+1})}{u'(d_t)}, R_t^s \right),$$

$$\begin{aligned} E_t(R_t^s) &= R_t - R_t C_t \left(\beta \frac{u'(d_{t+1})}{u'(d_t)}, R_t^s \right), \\ &= R_t - \frac{u'(d_t)}{\beta E_t(u'(d_{t+1}))} C_t \left(\beta \frac{u'(d_{t+1})}{u'(d_t)}, R_t^s \right), \\ &= R_t - \frac{C_t(u'(d_{t+1}), R_t^s)}{E_t(u'(d_{t+1}))}. \end{aligned}$$

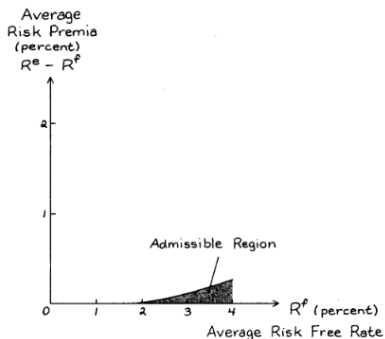
- ▶ The expected return on stocks equals the return on the risk-free bond plus the risk-premium, which is $-\frac{C_t(\cdot, \cdot)}{E_t(\cdot)}$.

Equity premium: a puzzle?

- ▶ If the covariance $C_t(\cdot, \cdot)$ is negative, which we normally expect, there is an equity premium.
- ▶ Interpretation
 - ▶ The most desirable assets yield well when marginal utility is high ($C_t(\cdot, \cdot) > 0$). Risk-aversion means that agents prefer assets that act like insurance.
 - ▶ Investors are willing to sacrifice return if $C_t(\cdot, \cdot) > 0$, and they will demand higher returns if $C_t(\cdot, \cdot) < 0$.
- ▶ Mehra and Prescott (1985): the observed equity premium is difficult to reconcile with this formula.

Equity premium: a puzzle?

- ▶ Mehra and Prescott (1985): the observed equity premium is difficult to reconcile with this formula.



- ▶ But most other assets have $> 10\%$ excess returns.

Conclusion

- ▶ Homework due tonight.
- ▶ No class next Tuesday (2/25). Instead, work on HW3.
- ▶ For Thursday (2/27), review lecture 11 on the stochastic neoclassical growth model.
- ▶ Hope to resume class on 3/4 with lecture 10.
- ▶ Midterm in about a month.

