

# Quantitative Macro-Labor: Expectations and Belief Formation

Professor Griffy

Fall 2024

# Announcements

- ▶ Briefly review beliefs and rational expectations.
- ▶ Show a model of inequality derived from beliefs.
- ▶ Presentation schedule

# Schedule

- ▶ Thursday (11/21)? 1 end of class
- ▶ Tuesday (12/3)? 3 people (25 mins each)
- ▶ Thursday (12/5)? 3 people (25 mins each)

# Portfolio Problem

- ▶ Consider a portfolio allocation problem in which an agent chooses between one of two assets:
  1. Asset  $a_{safe}$  offers a return of  $r_F$ , which is known with certainty.
  2. Asset  $a_{risky}$  offers a return of  $r_R$ ,  $r_R > r_F$  with probability  $\lambda$  and 0 with probability  $(1 - \lambda)$ ,
- ▶ Agents maximize a static portfolio problem:

$$V(m) = \max_{a_{safe}, a_{risky}} \lambda u(c'_H) + (1 - \lambda)u(c'_L) \quad (1)$$

$$c'_H = (1 + r_F)a_{safe} + (1 + r_R)a_{risky} \quad (2)$$

$$c'_L = (1 + r_F)a_{safe} + a_{risky} \quad (3)$$

$$m = a_{safe} + a_{risky} \quad (4)$$

# Parameter Uncertainty

- ▶ What if  $\lambda$  is type-specific?
- ▶ Asset  $a_{risky}$  offers a return of  $r_R$ 
  1. with probability  $\lambda_H$  for a high-type and  $\lambda_L$  for a low-type
  2. and 0, with probability  $(1 - \lambda_H)$  for a high-type and  $(1 - \lambda_L)$  for a low-type.
  3. Agents endowed with prior belief that they are high type,  $\theta^i \in [0, 1]$
- ▶ Prior beliefs drawn from uniform distribution  $g(\theta^i) \sim U(0, 1)$ .
- ▶ Belief distribution is agent-specific:  $h(\theta^i)$  may differ based on history. Initially  $h(\theta^i) = g(\theta^i) = \theta$
- ▶ For simplicity ignore bandit problem.

## Signal Extraction

- ▶ Bayes theorem:

$$h(\theta'|c') = \frac{f(c'|\lambda)b(\lambda|\theta)g(\theta)}{f(y)} \quad (5)$$

- ▶ Binomial likelihood:

$$f(c_H|\lambda) = \lambda^{1_{c'=c_H}}(1-\lambda)^{1-1_{c'=c_H}} \quad (6)$$

- ▶ Binomial likelihood:

$$b(\lambda|\theta) = \lambda_H \text{ with prob. } \theta = \lambda_L \text{ with prob. } (1-\theta) \quad (7)$$

- ▶ Prior Distribution ( $U(0, 1)$ ):

$$g(\theta) = \theta, \theta \in [0, 1], 0 \text{ else} \quad (8)$$

- ▶ Updating ( $f(y) = 1$ ):

$$h(\theta'|c' = c'_H) = \frac{\lambda_H \theta}{\lambda_H \theta + \lambda_L (1-\theta)} \quad (9)$$

$$h(\theta'|c' = c'_L) = \frac{(1-\lambda_H)\theta}{(1-\lambda_H)\theta + (1-\lambda_L)(1-\theta)} \quad (10)$$

## Discussion

- ▶ Updating ( $f(y) = 1$ ):

$$h(\theta' | c' = c'_H) = \frac{\lambda_H \theta}{\lambda_H \theta + \lambda_L (1 - \theta)} \quad (11)$$

$$h(\theta' | c' = c'_L) = \frac{(1 - \lambda_H) \theta}{(1 - \lambda_H) \theta + (1 - \lambda_L) (1 - \theta)} \quad (12)$$

- ▶ Thoughts about this updating:
  - ▶ What could be subjective here?
  - ▶ What could be general equilibrium here?

# Beliefs and Affirmative Action in Employment

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Summer 2023



# Motivation

- ▶ Large and persistent earnings gaps between Black and White.
- ▶ Large difference explained by occupational choice (58% CPS).
- ▶ Black teens who **believe** they will be **discriminated** against
  - ▶ aspire to lower-pay, less-prestigious occupations,
  - ▶ with more Black representation (this paper).
- ▶ Changing representation change beliefs & raise human capital?
- ▶ Can Affirmative Action permanently reduce racial inequality?

## This paper

- ▶ Develop a model with two-sided beliefs and learning about
  - ▶ discrimination and productivity.
- ▶ What we find (baseline, w/o affirmative action):
  - ▶ Historical underrepresentation leads Black workers to anticipate discrimination.
  - ▶ This causes underinvestment in human capital.
  - ▶ The average Black candidate is less qualified, causing firms to statistically discriminate.
  - ▶ Causes more pessimism among new Black workers and repeats.
  - ▶ Slow convergence to steady-state.
- ▶ Black income 75% of White income ( $\approx$  same as data)

## This paper

- ▶ Develop a model with two-sided beliefs and learning about
  - ▶ discrimination and productivity.
- ▶ After Affirmative Action implemented:
  - ▶ Increase in employment causes optimistic beliefs among next generation of Black workers.
  - ▶ Recognition that most discrimination is statistical & and can be overcome by investment.
  - ▶ Causes an increase in human capital investment, and reverses cycle.
  - ▶ Rapid convergence.
- ▶ Black income 89% of White income *after Affirmative Action*.
- ▶ Although there are short-term costs (underqualified, reinforce stereotypes, etc.), they are outweighed by long-term gains.

# Model Overview

- ▶ During current period
  - ▶ workers invest in human capital ( $z$ ) & apply for jobs;
  - ▶ firms receive set of applications & hire “best” applicant.

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  - ▶ firms update beliefs about productivity by race ( $f(z|r)$ ) based on workers interviewed & hired.

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- ▶ Discrimination:
  - ▶ Some firms are perm. taste-based discriminators (measure  $\mu$ ), but identities unknown.
  - ▶ all firms statistically discriminate based on history ( $f(z|r)$ ),
  - ▶ workers believe measure  $\hat{\eta}$  are taste-based discrim., but can't distinguish sources.

# Worker Static Optimization

- ▶ Workers initial state:
  - ▶ Race,  $r$ , innate ability  $q \sim LN(\mu_Q, \sigma_Q)$
  - ▶ Employment beliefs  $\hat{P}(e|z, r; \hat{\eta})$
- ▶ Workers problem:
  - ▶ Make costly human capital investment decision,  $z$ , ( $\frac{\partial c(z)}{\partial q} < 0$ ).
  - ▶ Apply to high prestige job with cost  $\nu \sim Gumbel$ ,
  - ▶ believe prob  $\hat{P}$  of job offer.

# Firm Static Optimization

- ▶ Firm state: Beliefs  $f_i(z|r) \sim \text{Beta}(\alpha_r, \beta_r)$  with prior  $p(\alpha_r, \beta_r)$ .
- ▶ Firm's problem: Receive  $H \sim \exp(\lambda)$  applications,
  - ▶ get noisy signal  $y_j = \ln(z_j) + \epsilon_j$ ,  $\epsilon \sim N(0, \sigma_\epsilon)$
  - ▶ assign score  $s_i(y, r, T, f) = E[z|y, r, f] - 1_{D}^B \gamma + 1_{AA}^B \xi$
  - ▶ And hire  $\max\{s_1, \dots, s_{H_i}\}$ .
- ▶ Labor market resolution:
  - ▶ of  $N$  firms,  $e^B$  hire Black workers and  $e^W$  hire White.



## Workers updating beliefs

- ▶ What they know:

- ▶ Aggregate hiring outcomes by race,  $e_j^B, e_j^W \forall j \leq t$ ,

- ▶ Pred. prev. emp.  $\hat{e}_j^r(\hat{\eta}) = \int_q \hat{P}_j(e|z, r; \hat{\eta}) \frac{\partial z_j(q, r)}{\partial q} dF(q) \forall j < t$

- ▶ Update  $\hat{\eta}$  to minimize difference between this history:

$$\min_{\hat{\eta}} \sum_{j=0}^{t-1} \left( \frac{\hat{e}_j^r(e|B; \hat{\eta})}{\hat{e}_j^r(e|B; \hat{\eta}) + \hat{e}_j^r(e|W; \hat{\eta})} - \frac{e_j^B}{e_j^B + e_j^W} \right)^2 \quad (13)$$

- ▶ where  $\hat{e}^r(e|B; \hat{\eta}) = \hat{\eta} \hat{e}^r(e|B, D) + (1 - \hat{\eta}) \hat{e}^r(e|B, N)$

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- ▶ Need  $\hat{P}_t(e|z, r; \hat{\eta})$ . Approximate firm problem:

- ▶ Calc. scores w/ unbiased beliefs:  $s(y, r, T) = y - 1_D^B \gamma + 1_{AA}^B \xi$

- ▶ Calc.  $\hat{P}(\cdot; T) = P(s_j = \max\{s_1, \dots, s_H\} | z, r, T)$  (signal)

- ▶ Est.  $e_t^r(D)$  and  $e_t^r(N)$  and use  $\hat{P}$  with weight  $\hat{\eta}$ .

## Firms updating beliefs

- ▶ Firm information carried over from previous period:
  - ▶ Learned true productivity  $z$  of hired worker (and their race).
  - ▶ prior,  $p(\alpha_r, \beta_r)$  (over  $f(z|r) \sim \text{Beta}(\alpha_r, \beta_r)$ ).
- ▶  $\rightarrow$  sample of own hires/interviews  $X = \{\{\neg y, \neg r\}, (y, z, r)\}$ .

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- ▶  $\rightarrow$  sample of own hires/interviews  $X = \{\{\neg y, \neg r\}, (y, z, r)\}$ .
- ▶ they calc.  $p(X|\alpha, \beta)$  for parameter space of  $(\alpha, \beta)$  & update:

$$p(X|\alpha, \beta) = \prod_{i=1}^H \underbrace{\left( \int \frac{e^{-\frac{1}{2} \left( \frac{y_i - \ln(z)}{\sigma} \right)^2}}{2\sqrt{\pi}\sigma} f(z|r) dz \right)^{1 - 1_{s_i=\hat{s}}}}_{\text{noisy signal}} \underbrace{f(z|r)^{1_{s_i=\hat{s}}}}_{\text{observed}} \quad (14)$$

- ▶ & use  $p(X|\alpha, \beta)$  & update using Bayes' Rule:

$$p(\alpha, \beta|X) = \underbrace{p(X|\alpha, \beta)}_{\text{likelihood}} \times \underbrace{p(\alpha, \beta)}_{\text{prior}} \quad (15)$$

- ▶ to form  $f(z|y, r) = \int_{\alpha \times \beta} f(z|y, r; \alpha, \beta) p(\alpha, \beta|X) d\alpha d\beta$

# Findings

- ▶ Set-up:
  - ▶ Explore time series of 20 cohorts.
  - ▶ Initial conditions: fix  $\hat{\eta}$  and let  $Z^0(q, r)$  and  $\hat{P}(e|z, r; Z^0, \hat{\eta})$  converge.
- ▶ Explore the mechanisms:
  - ▶ How do observed outcomes affect worker and firm beliefs?
  - ▶ How do those beliefs affect subsequent decisions?
- ▶ Then impose Affirmative Action policy (details after mech.)

# Human capital investment (1st Cohort)

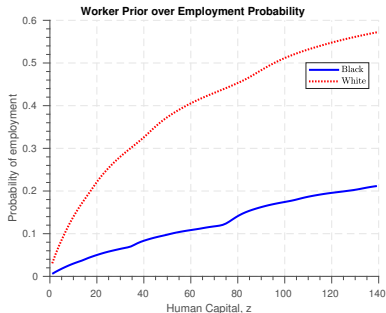


Figure: Beliefs about employment probabilities given  $z$  by race.

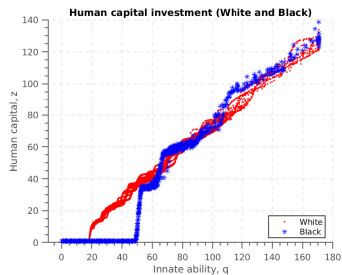


Figure: Human capital investment decisions ( $z$ ) by innate ability ( $q$ ) and race.

- ▶ Lower employment probability  $\rightarrow$
- ▶ Less investment over key range (98th pctile).

# Factors influencing hiring decisions (1st Cohort)

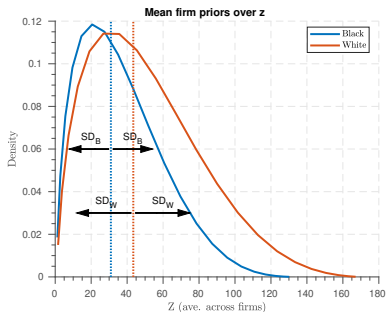


Figure: Average firm beliefs over human capital ( $z$ ) by race.

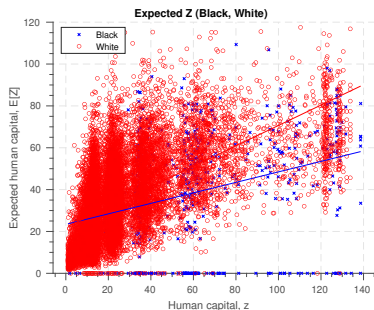


Figure: Black and White  $z$  and  $E[z]$

- ▶ Firms believe avg. Black worker less qualified, no weight on high  $z$ .
- ▶ Bias ( $E[z] - z$ ) much larger for Black than White.

# Discrimination beliefs and employment beliefs (1st Cohort)

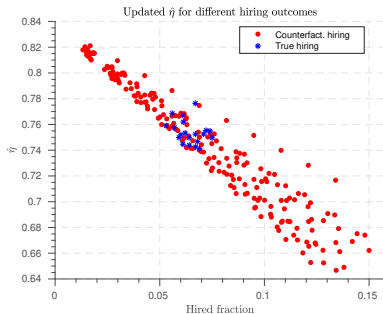


Figure: Updated  $\hat{\eta}$  under different hiring.

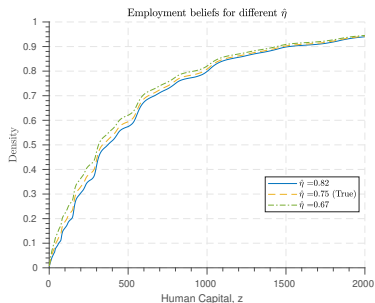


Figure: Black employment beliefs under different  $\hat{\eta}$ .

- ▶  $\hat{\eta}$  varies negatively ( $\uparrow$  emp.  $\downarrow$   $\hat{\eta}$ ) with hiring outcomes.
- ▶ Lower  $\hat{\eta}$   $\rightarrow$  more optimistic  $\hat{P}$ .



# Human Capital Investment

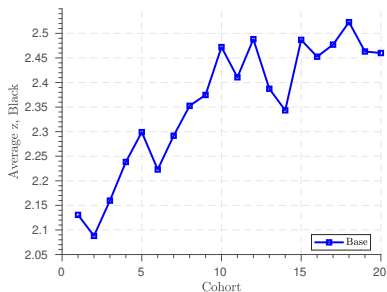


Figure: Human Capital z (Black)

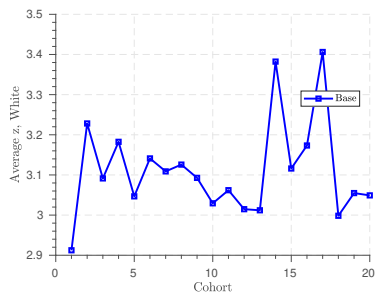


Figure: Human Capital z (White)

- ▶ Higher levels of investment for White workers.
- ▶ Both vary over time, inversely related.

# Policy Experiment

- ▶ Affirmative Action policy
  - ▶ *One cohort*, Black worker scores  $\uparrow$  by  $\zeta = \gamma$  (ad-hoc).
  - ▶ Implemented *after* investment decisions by cohort.
- ▶ Two Affirmative Action stories to explore:

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  1. Short-term direct effects:
    - ▶ Do Black workers replace more qualified White workers?
    - ▶ Do less qualified Black workers cause firm to revise beliefs down?

# Policy Experiment

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  - ▶ *One cohort*, Black worker scores  $\uparrow$  by  $\zeta = \gamma$  (ad-hoc).
  - ▶ Implemented *after* investment decisions by cohort.
- ▶ Two Affirmative Action stories to explore:
  1. Short-term direct effects:
    - ▶ Do Black workers replace more qualified White workers?
    - ▶ Do less qualified Black workers cause firm to revise beliefs down?
  2. Post-AA indirect effects of more initial Black hires:
    - ▶ After AA period, do additional new hires change Black beliefs?
    - ▶ Does this lead to more Black human capital investment?
    - ▶ What happens to White workers with more competition?

# Initial Costs

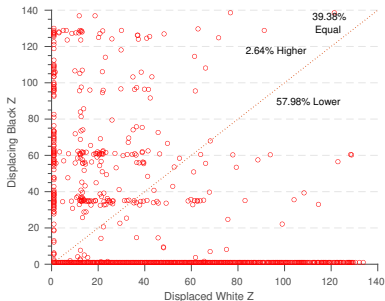


Figure: White workers displaced by Black workers newly hired

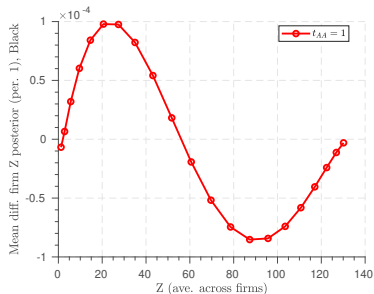


Figure: Difference in firm Z beliefs (Black)

- ▶ 58% of White workers displaced are more qualified.
- ▶ Posterior firm beliefs imply firms expect lower avg. Black z.

# Long-Run Benefits (2nd Cohort)

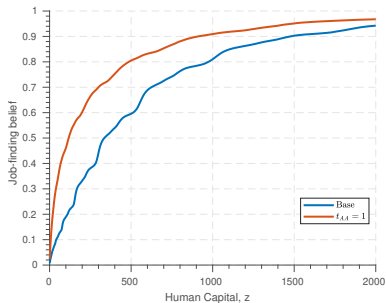


Figure:  $\hat{P}(e|z, B; Z^0, \hat{\eta})$  comparison.

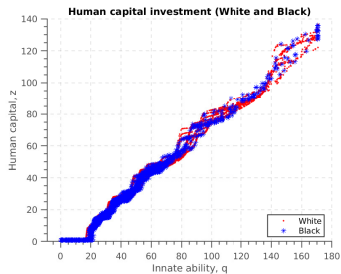


Figure: Human capital investment decisions ( $z$ ) by innate ability ( $q$ ) and race.

- ▶ For next cohort, large decline in  $\hat{\eta}$  (70pp, next slide)
- ▶  $\rightarrow$  upward revision in employment beliefs for all  $z$ .
- ▶ Investment mirrors White workers! ( ▶ baseline )

# Long-Run Benefits

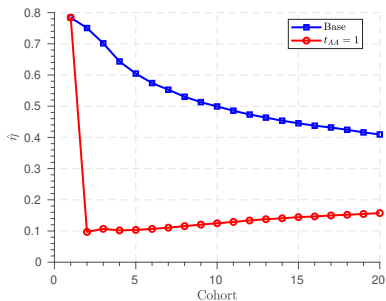


Figure:  $\hat{\eta}$  across cohorts

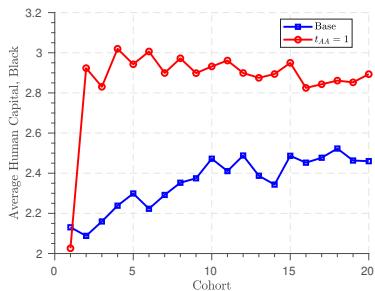


Figure: Human capital  $z$  (Black)

- ▶ Decrease in beliefs about discrimination persist.
- ▶ Large increases in Black  $z$  (averaged, including non-investment).

## Overall Effects

Value	Base	$t_{AA} = 1$
Ave. $\hat{\eta}$	0.5240	0.1610
Employed Z (Post AA)	18.627	18.913
Black Z Invest	2.359	2.865
White Z Invest	3.162	3.139
Black Employed Z (Post AA)	19.565	19.646
White Employed Z (Post AA)	18.536	18.819
Black High Prestige Emp. Rate	0.0530	0.0840
White High Prestige Emp. Rate	0.0910	0.0860
Average Black Income	1.857	2.127
Average White Income	2.480	2.387

- ▶ Racial income inequality:
  - ▶ Baseline: Black-White income ratio 75% ( $\approx$  same as data).
  - ▶ After Affirmative Action Policy: 89%.
  - ▶  $\approx$  share accounted for by occupational choice.
- ▶ Emp. z: up for both Black and White!



# Overview

- ▶ Constructed a model with
  - ▶ Endogenous worker beliefs about discrimination and employment;
  - ▶ Endogenous firm beliefs about worker productivity.
- ▶ Assessed the effects of Affirmative Action.
- ▶ Findings:
  - ▶ Affirmative Action may displace more qualified White workers, and negatively affect firm beliefs in short-run.
  - ▶ Has dynamic benefits: increases Black human capital investment by changing beliefs about employment prospects.
  - ▶ Overall positive effect on *both* Black and White investment.

## Next Time

- ▶ No class 11/26.
- ▶ Start presentations of final projects after Break.

# Discrimination and Aspirations

- ▶ Black youths who believe they face discrimination
  - ▶ aspire to less prestigious/lower pay occupations.

	Prestige Score of Career Aspiration in 1979	Aspired Occupational Income, 1970s
Black	7.0448**	4768.1849
	1.4598	2182.1669
Believes Discrimination will Affect Career=1	7.34**	13358.49***
	2.29	1681.90
Black × Believes Discrimination will Affect Career=1	-8.93***	-16090.41**
	1.35	4652.43
Test: $H_0 : \beta_1 + \beta_3 = 0$	-1.881***	-11322.22**
SE	0.260	2483.498
Observations	1296	1164

Clustered standard errors in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

- ▶ These careers have higher than average Black representation,
- ▶ and is realized in actual occupations at age 35. ([▶ link](#))
- ▶ More stats about beliefs and discrimination: ([▶ link](#)) ([▶ back](#))

# Discrimination and Outcomes

- ▶ Black youths who believe they face discrimination
  - ▶ enter occupations with higher Black representation and achieve less-prestigious careers.

	Percent of Black Workers in Aspired Career	Prestige of Age-35 Occupation
Black	-0.0063	-2.4247
	0.0031	1.3379
Believes Discrimination will Affect Career=1	-0.01***	2.09
	0.00	3.45
Black × Believes Discrimination will Affect Career=1	0.03**	-4.99
	0.01	2.26
Test: $H_0 : \beta_1 + \beta_3 = 0$	.024*	-7.419***
SE	0.010	1.178
Observations	1164	1293

Clustered standard errors in parentheses

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

▶ back

# Robustness: Same Occupation as Father

- ▶ Same specification, with

$$1_{\text{Aspired Father's Occ}} + 1_{\text{Black}} \times 1_{\text{Aspired Father's Occ}}$$

	Prestige Score of Career Aspiration in 1979 b/se	Aspired Occupational Income, 1970s b/se
Black	7.3014**	4842.5913*
	1.3964	1956.5340
Believes Discrimination will Affect Career=1	7.22*	13148.80***
	2.34	1881.27
Black × Believes Discrimination will Affect Career=1	-8.75**	-15848.36*
	1.58	5000.00
sameAspiredOccFather=1	-2.77	-3803.78
	1.21	4461.42
Black × sameAspiredOccFather=0	0.00	0.00
	.	.
Black × sameAspiredOccFather=1	-8.87*	-3407.10
	3.22	8357.90
Observations	1296	1164
Test: $0: \beta_{-1} + \beta_{-3} = 0$	-1.453***	-11005.771**
SE	0.240	3057.215
Test: $0: \beta_{-1} + \beta_{-5} = 0$	-1.572	1435.494
SE	3.640	9697.593

Clustered standard errors in parentheses

\* p < 0.1, \*\* p < 0.05, \*\*\* p < 0.01

▶ back

# Black Beliefs about Discrimination

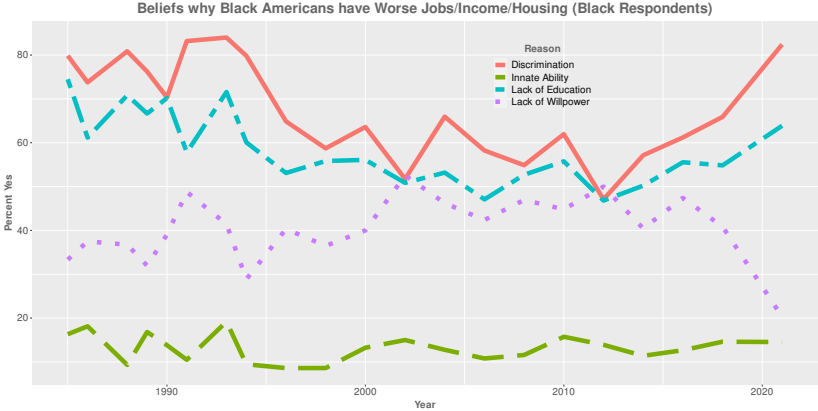


Figure: Black Beliefs

▶ Attribute differences to discrimination. [▶ back](#)

# Firm Beliefs about Black Workers

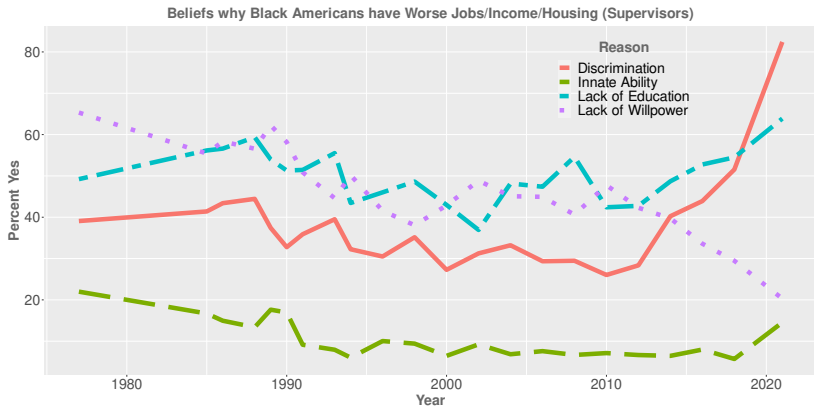


Figure: Supervisor Beliefs

- ▶ (implicit assumption: Supervisors have discretion over hiring)
- ▶ Other evidence: audit studies, other responses to GSS. [▶ back](#)

## Worker's human capital decision

- ▶ Endowed with race,  $r$ , innate ability,  $q \sim LN(\mu_Q, \sigma_Q)$ , and
- ▶  $\hat{\eta}$ : common belief about measure of discriminators,
- ▶  $\hat{P}$ : Prob(emp | own  $z$ , other  $z$ , discrim. beliefs) (fixed pt).
- ▶ Live for one period (think cohort).
- ▶ Investment decision:

$$V_I(z, r, \hat{P}; \hat{\eta}) = \max_z \left\{ E[V_A(z, r, \hat{P}; \hat{\eta})] - \frac{z^2}{2q} \right\} \quad (16)$$



## Worker's application decision

- ▶ Can pay pay  $\nu \sim \text{Gumbel}(\sigma_\nu)$  to apply for high-prestige job;
- ▶ If not hired or don't apply, apply for "humble" job.
- ▶ High-prestige application decision:

$$V_A(z, r, \nu, \hat{P}; \hat{\eta}) = \max \left\{ \hat{P}(e|z, r; \hat{\eta})z + (1 - \hat{P}(e|z, r; \hat{\eta}))V_L + \nu, V_L \right\} \quad (17)$$

- ▶  $V_L = P_L z_L + (1 - P_L)b$  (think service sector)

## Worker's application decision

- ▶ Can pay pay  $\nu \sim \text{Gumbel}(\sigma_\nu)$  to apply for high-prestige job;
- ▶ If not hired or don't apply, apply for "humble" job.
- ▶ High-prestige application decision:

$$V_A(z, r, \nu, \hat{P}; \hat{\eta}) = \max \left\{ \hat{P}(e|z, r; \hat{\eta})z + (1 - \hat{P}(e|z, r; \hat{\eta}))V_L + \nu, V_L \right\} \quad (17)$$

- ▶  $\hat{P}(e|z, r; \hat{\eta})$  depends on beliefs about discrimination
  - ▶ Discrim. beliefs  $\hat{\eta} \uparrow \rightarrow \hat{P}(e|z, B; \hat{\eta})$  beliefs  $\downarrow$
  - ▶  $\rightarrow \hat{\eta} \uparrow \rightarrow z^* \downarrow$

## Firm's hiring decision

- ▶ Hire to maximize exp. output ( $z$ ), net of racial preferences
- ▶ Filter signals to find best candidate.
- ▶ Receive application, get signal of worker productivity:

$$y = \ln(z) + \epsilon, \epsilon \sim N(0, \sigma_\epsilon) \quad (18)$$

- ▶ Each applicant receives a score:

$$s(y, r, T) = E[\tilde{z}|y, r] - 1_D^B \gamma + 1_{AA}^B \zeta. \quad (19)$$

- ▶  $1_D^B$ : taste-based discriminator (reduces Black score by  $\gamma$ );
- ▶  $1_{AA}^B$ : affirmative action policy (increases Black score by  $\zeta$ );
- ▶  $E[\tilde{z}|y, r]$ : Beliefs (may be biased) about the worker prod.

## Firm's hiring decision

- ▶ Receive application, get signal of worker productivity:

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- ▶  $E[\tilde{z}|y, r]$  depends on history of hires and observed signals:

$$E[\tilde{z}|y, r] = \int z f(z|y, r) dz = \int z \frac{f(y|z, r) f(z|r)}{f(y|r)} dz \quad (20)$$

- ▶ Each firm hires  $\max\{s_1, \dots, s_H\}$ .
- ▶ Produce worker's true  $z$

## Workers updating beliefs I

- ▶ Construct score for taste and non-taste discrim. ( $\epsilon$  unknown):

$$s(y, r, T) = z + \epsilon - 1_D^B \gamma + 1_{AA}^B \zeta.$$

- ▶ Calc.  $\hat{P}$  given dec. rules  $Z^0$ . Def.  $F(\cdot)$  CDF of  $\epsilon$ .

$$\tilde{P}(e|z, r; Z^0, T) =$$

$$\prod_{i=2}^H \left[ \int_{\epsilon_i} \int_{q_i} \sum_{r_i=B}^W \underbrace{\left( \int_{s(Z^0(q_i, r_i) + \epsilon_i, r_i, T)}^{s_{\max}} \right)}_{\text{app. } i \text{ score}} \overbrace{f(y(s, r) - \ln(z)) \left| \frac{\partial y}{\partial s(y, r)} \right| ds}_{\text{prob. 1st score higher}} f(\epsilon_i) p(r_i) dQ(q_i) d\epsilon_i \right].$$

(21)

## Workers updating beliefs I

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$$\tilde{P}(e|B; Z^0, T) = \int_{\underline{q}}^{\bar{q}} \tilde{P}(e|z(q, B), B; Z^0, T) dQ(q)$$

- ▶ Form predicted hiring given  $\hat{\eta}$  (scale by num. Black workers):

$$\tilde{P}(e|B; Z^0, \hat{\eta}) = \hat{\eta} \tilde{P}(e|B, D) + (1 - \hat{\eta}) \tilde{P}(e|B, N)$$

# Calibration Results

Parameter	Value	Comment
$\sigma_Q$	1.38	SD of Innate Ability Dist.
$\sigma_\epsilon$	0.816	SD of Signal Noise
$\sigma_\nu$	1.38	SD of Application Taste Shock
$\gamma$	50	Taste-Based Discrimination Score Penalty
$Pr(e_L)$	0.788	Low Prestige Employment Probability
$\mu$	0.221	Supervisor Responses about Lower Black Ability (GSS, 1977)
$\hat{\eta}_0$	0.784	Black Responses about Labor Market Discrimination (GSS, 1985)
$b$	0.4	Approx UI Replacement Rate (US)
$z_L$	1.00	Normalization
$\zeta$	50	Assumption
$\mu_Q$	1.00	Normalization

▶ back

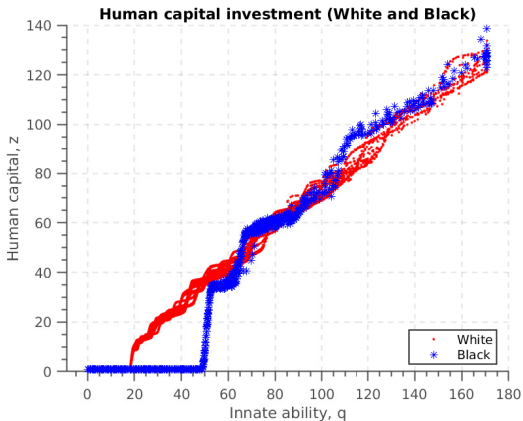
# Calibration Fit

Moment	Data	Model
Black-to-White Earnings Ratio	0.6273	0.6591
95/5 Earnings Ratio (Pooled)	5.8203	5.8454
Black Unemployment Rate	0.0597	0.0401
White Unemployment Rate	0.0285	0.0390
Black Employment Rate (25th AFQT Pctile)	0.9100	0.9533
Black Employment Rate (75th AFQT Pctile)	0.9600	0.9591
White Employment Rate (25th AFQT Pctile)	0.9600	0.9622
White Employment Rate (75th AFQT Pctile)	0.9900	0.9595
Ratio of Black-to-White High Prestige Employment Rates	0.5623	0.5149

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# Human capital investment Baseline



**Figure:** Human capital investment decisions ( $z$ ) by innate ability ( $q$ ) and race.

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