# Quantitative Macro-Labor: General Equilibrium Search and Matching

Professor Griffy

UAlbany

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# Announcements

- Today: Estimating a search model.
- Two more lectures:

# Estimation Techniques

- Broadly two categories of estimation techniques.
- Limited information techniques:
  1. Simulated method of moments;
  2. Indirect inference;
  3. Calibration (just SMM).
- Key idea: specify set of moments that model should be good at matching.
- Calibration vs. SMM: more about approach to robustness.
- Full information methods:
  1. Maximum likelihood;
  2. Bayesian estimation.
- Comparison?
- Excellent reference: DeJong and Dave (2011) "Structural Macroeconometrics"

# Limited Information Estimation

- Basic idea:
  - Choose moments that model should be able to match.
  - What are some moments?
  - Means, variances.
- Two approaches:
  - Derive conditions and estimate parameters.
  - Simulate data and match moments.

# "Pre-Calibrated"

- AKA preset, sometimes mistakenly called calibrated.
- What people think it means: pick some parameters from other macro papers.
- When you do this, it is called a "numerical exercise."
- Fine for first pass, or theory paper, but not for quantitative paper.
- What pre-calibrated really means:
  - Parameters estimated in another model.
  - Model in your paper is *very* similar.
  - "Preset" parameters that don't affect what differentiates your model.

# External Calibration

- Some model features are directly observable in the data.
- Typically, linear function $f(x, y)$, or non-linear function that can be linearized.
- Cobb-Douglas production function:

$$Y = F(K, L) = zK^{\alpha}L^{1-\alpha} \tag{1}$$

- Need to estimate? $\alpha$, and $z_t$ as well!

$$ln(Y) = ln(z) + \alpha ln(K) + (1-\alpha)ln(L) \tag{2}$$

- Run regression:

$$ln(Y_t) = \beta_1 ln(K) + \beta_2 ln(L) + \epsilon_t \tag{3}$$

- Then $\alpha = \frac{\beta_1}{\beta_1+\beta_2}$, $ln(z_t) = \hat{\epsilon}_t$

# External Calibration

- This is *much* simpler than other estimation techniques.
- Take this approach when you can.
- Other applications:
  1. Income processes we saw early in class.
  2. Depreciation (human capital or physical capital).
- Others?

# Simulated Method of Moments

- Basic idea, simulate model, compare outcomes to data.
- Outline:
  - Define set of unconditional moments in data.
  - Pick initial parameter values.
  - Solve and simulate model, generate same unconditional moments with model data.
  - Compare moments, calculate squared residuals.
  - Guess new parameter values.
- Complications:
  - Need a routine to pick new parameters.

# Simulated Method of Moments

- Define set of empirical targets $h(z_t)$
- Define theoretical counterparts $h(y_t, \theta)$
- Goal, find $\theta$ s.t. $E[h(y_t, \theta)] = E[h(z_t)]$.
- Define sample analogue $g(Z, \theta)$:

$$g(Z, \theta) = \frac{1}{T} \sum_{i=1}^{T} h(z_t) - \frac{1}{N} \sum_{i=1}^{N} [\frac{1}{T} \sum_{t=1}^{T} h(y_t, \theta)] \tag{4}$$

- Then the objective function is given by

$$\min_{\theta} \Gamma(\theta) = g(Z, \theta)' \times W \times g(Z, \theta) \tag{5}$$

- Always tricky to pick correct weighting matrix $W$.

# Practical Implementation

- Note that simulated moments involve both $T$ and $N$.
- The reason: each iteration, you run the model $N$ times.
- Because you need to average out the randomness of simulations.
- Then, usually minimize the squared residual with some weighting matrix.
- Often, people use identity matrix or inverse of empirical variance; both are not efficient in the statistical sense.

# Calibration

- Two types of calibration:
  - Derive conditions and estimate parameters.
  - Simulate data and match moments.
- First is "external calibration"
- Second is SMM, but without recovering standard errors of parameters.
- Good reference: Cooley (1995).

# Indirect Inference

- Simulated method of moments with conditional moments.
- Instead of matching means, variances, etc., match regression and other reduced-form moments.
- Define an "auxiliary model":
  - Empirical specifications that are easy to compute.
  - Can be easily simulated by the model.
  - Capture essential elements of equilibrium.
- Important: auxiliary model *does not need to map one-to-one with structural parameters!*
- That is, it can be a "perturbed" version of your model.

# Indirect Inference

- Two steps:
  - Define sample analog of empirical moments $\delta(z_t)$, $\delta(Z) = \arg\max_\delta \Delta(z, \delta)$
  - Then theoretical analog: $\delta(Y, \theta) = \arg\max_\delta \Delta(Y, \delta)$
- In words: use $\theta$ to match a vector of parameters $\delta$.

$$g(Z, \theta) = \delta(Z) - \frac{1}{S} \sum_{i=1}^{S} \delta(Y^i, \theta) \qquad (6)$$

- Simulate each iteration $S$ times.

# Objective Function

- There are multiple ways to define the objective function.
- Intuitive way:

$$\min_{\theta} \Gamma(\theta) = [\Delta(Z) - \delta_S(Y,\theta)]' \times W \times [\Delta(Z) - \delta_S(Y,\theta)] \tag{7}$$

- Also: gaussian objective.

# Full Information Methods

- Variations on maximum likelihood.
- May cover next Tuesday (or see online handout).
- Useful when model involves distributions (productivity, wages, etc.)
- Good references:
  - Chris Flinn's work.
  - Rasmus Lentz's work.
  - "European Search" group (Postel-Vinay, Robin, etc.)

# The DMP Model ("Ch. 1 of Pissarides (2000)")

- Agents:
  1. Employed workers;
  2. unemployed workers;
  3. vacant firms;
  4. matched firms.
- Linear utility ($u = b, u = w$) and production $y = p > b$.
- Matching function:
  1. Determines *number* of meetings between firms & workers.
  2. Args: levels searchers & vacancies ($U = u \times L, V = v \times L$)
  3. Constant returns to scale ($L$ is lab. force):

$$M(uL, vL) = uL \times M(1, \frac{v}{u}) = uL \times p(\theta) \tag{8}$$

  4. where $\theta = \frac{v}{u}$ is "labor market tightness"
  5. Match rates:

$$\underbrace{p(\theta)}_{Worker} = \theta \underbrace{q(\theta)}_{Firm} \tag{9}$$

# Equilibrium Objects

- Three key equilibrium objects:
  1. Wages;
  2. unemployment;
  3. $\theta = \frac{v}{u}$ (vacancies).
- How we determine each of these is largely a modeling decision.
- Steady-state: pin down unemployment via flow equation.
- Free-entry: Assume that firms always post vacancies so that free entry binds.
- Wages: Assume that wages are determined by a surplus- (profit) sharing rule.

# Steady-State Unemployment

- Flow of unemployment:

$$\dot{u} = \delta(1-u) - p(\theta)u \tag{10}$$

- Steady-state:

$$0 = \delta(1-u) - p(\theta)u \tag{11}$$

$$p(\theta)u = \delta(1-u) \tag{12}$$

$$u = \frac{\delta}{\delta + p(\theta)} \tag{13}$$

- Same as McCall with $\alpha = p(\theta)$.
- (Note: no heterogeneity & $p > b \rightarrow$ all wages accepted.)

# Free Entry

- Free entry $V = 0$:

$$rJ(w) = (p - w) + \delta[\cancel{V} - J(w)] \tag{14}$$

$$(r + \delta)J(w) = (p - w) \tag{15}$$

- Vacancy creation condition (i.e., free entry imposed):

$$q(\theta) = \frac{\kappa}{E[J(w)]} \tag{16}$$

$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)} \tag{17}$$

$$\theta = q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right) \tag{18}$$

- Thus, mapping between wages and $\theta$. 1 equation, 2 unknowns.
- Need equation to determine wages in equilibrium.

# Wage Determination

- Note that $\beta S(w) = [W(w) - U]$

$$(1-\beta)(w-b) = \beta(p - w - \delta J(w)) \tag{19}$$
$$+ (1-\beta)(p(\theta)+\delta)\beta S(w) \tag{20}$$

- And $(1-\beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1-\beta}$

$$(1-\beta)(w-b) = \beta(p - w - \delta J(w)) \tag{21}$$
$$+ (1-\beta)(p(\theta)+\delta)\beta \frac{J(w)}{1-\beta} \tag{22}$$
$$w = (1-\beta)b + \beta p + p(\theta)\beta J(w) \tag{23}$$

- Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1-\beta)b + \beta p + \beta\theta\kappa \tag{24}$$

# Estimation

- What parameters do we need to estimate/pick?
  - $\delta$: (exogenous) separation rate.
  - $b$: unemployment utility.
  - $\beta$: bargaining power.
  - $\kappa$: vacancy creation cost.
  - $r$: discount rate.
  - utility function (linear).
  - Cobb-Douglas Matching: $M(u, v) = Au^{\alpha}v^{1-\alpha}$
- What can we externally calibrate?
  - $\delta$: E-U flows.
  - $\alpha$: U-E flows + vacancy & unemployment rate.
  - Set $A = 1$ (maybe).
  - $r$: choose frequency (weekly, monthly, etc.) and pick interest rate (i.e., $\beta_{Discount} = \frac{1}{1+r}$)

# Estimation

- What is tricky to calibrate?
  - *b*: what is unemployment utility?
  - $\kappa$: what is the cost of opening a vacancy?
  - $\beta$: what is "bargaining power"?
- Important question: should we target equilibrium or most closely associated data?
- i.e., should $\kappa$ target estimates of the cost of posting a vacancy? Or should we target wages or another equilibrium object?
- Not obvious. Argument for model's validity is stronger the more directly you can point to a target.

# Using the right data

- Another important consideration: are you using the right data series?
- i.e., if your model doesn't have growth, you can't target the time series of GDP.
- Why? because your model isn't equipped to match it.
- Filtering:
  - Imagine time series has two components: trend and cycle.
  - De-trend data using HP-filter.
- Some series don't have trends: unemployment.
- Cooley (1995) is a really good reference for thinking about these issues.

# Shimer, 2005

- Influencial paper that really walks through sensible approaches to calibration.
- Some preliminaries:
  - Adds aggregate shocks, i.e., not steady state equilibrium.
  - Discrete time version of model.
- Calibration approach:
  - Target/set parameters to sensible values.
  - Do robustness checks with other parameters/assumptions.

# Shimer, 2005

- What is $b$?
- Shimer takes a very literal interpretation of $b$: sets $b$ to be the replacement rate $b = 0.4$).
- Bargaining power $\beta$? Assume that "Hosios Condition" holds.
- Hosios condition: $\beta = \alpha$, i.e., bargaining power equals elasticity of matching function.
- Most of second half of paper: robustness checks with alternate assumptions.
- Leads to "Shimer Puzzle": search models can't address business cycle fluctuations.

# Flinn, 2005

- Paper that addresses the minimum wage.
- Key parameter: worker's bargaining power.
- He takes a literal interpretation: sets $\beta =$ fraction of worker salaries out of total revenue at a large firm (McDonald's).
- Compares with Hosios Condition: very different outcomes!

# Conclusion

- Two ways to approach quantitative macro:
  - Seek permission: look for empirical regularities and write down model to try and explain them.
  - Ask forgiveness: write down model and then look for empirical regularities consistent with equilibrium.
- Both are valid ways to approach quantitative macro, and both can involve sunk costs.
- Final due date for project 2? Sometime around Dec 12th.