Quantitative Macro-Labor: Heterogeneous Agent Models

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Announcements

- Today: Continue heterogeneous agent models.
- First: Huggett and Aiyagari.
- Empirical regularities presentations due next Tuesday.
- Presentations also next Tuesday.
- Note: final project is solving a model that explains your empirical regularity.
- Will be presentations last two days of class.

Thinking about Uncertainty in Macroeconomic Models

- Typical assumptions in macroeconomics are a convex combination of
  1. certainty equivalence:

$$u'(\bar{c}_{i,t}) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \underbrace{u'(\bar{c}_{i,t+1})}_{Closer\ to\ Linear}] \tag{1}$$

  2. linearized decision rules:

$$\sum_{i=1}^{N}((1 + r_{t+1})a_{i,t+1} + w_{i,t+1} - c_{i,t+1} - a_{i,t+2}) = 0 \tag{2}$$

$$\sum_{i=1}^{N}((1 + r_{t+1})\beta_a \hat{S}_{i,t+1} + \beta_w(\hat{S}_{i,t+1}) - \beta_c \hat{S}_{i,t+1} - \beta_a \hat{S}_{i,t+2}) = 0 \tag{3}$$

- Trick in Krusell-Smith: assume that workers make a linear prediction about prices in the future.
- i.e., workers use OLS to predict future prices.

Heterogeneous Agent Models

- Workers change their behavior in response to uncertainty.
- First wave of heterogeneous agent models: how do aggregates change when *individual idiosyncratic* uncertainty is uninsurable.
- In other words: when agents must accumulate *precautionary savings* to insure against income shocks.
- Key "first wave" papers (no particular order):
  - Huggett (1993): Incomplete markets exchange economy with GE interest rate.
  - Imrohoroglu (1989): Individual and aggregate uncertainty with fixed interest rate.
  - Aiyagari (1994): Incomplete markets production economy with GE interest rate.
  - Bewley (1986): Individual uncertainty with fixed interest rate.
- Krusell and Smith (1998): individual and aggregate uncertainty with GE interest rate.
- Do this using an approximation to the aggregate evolution of capital.

Heterogeneous Agent Models

- We can write a generic worker's problem as

$$\max_{\{c_t, i_t, l_t\}_{t=0}^{\infty}} E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4}$$

$$\text{s.t. } c_t + i_t \leq r_t a_t + w_t l_t \tag{5}$$

$$a_{t+1} = (1-\delta) a_t + i_t \tag{6}$$

$$a_{t+1} \geq \underline{a}_t \tag{7}$$

$$w_t \sim F \tag{8}$$

$$c_t \geq 0, l_t \geq 0, a_0 \text{ given} \tag{9}$$

- How we deal with prices r_t, w_t and choices c_t, i_t, l_t is central to equilibrium.

Recursive Formulation

- Can be written as

$$
\begin{aligned}
V(a) &= u(c) + \beta E[V(a')] \quad (10)\\
\text{s.t. } c + i &\leq ra + wl \quad (11)\\
a' &= (1-\delta)a + i \quad (12)\\
a' &\geq \underline{a} \quad (13)\\
w &\sim F \quad (14)\\
c \geq 0, l &\geq 0, a_0 \text{ given} \quad (15)
\end{aligned}
$$

- Under fairly general conditions, this inherits same properties as non-stochastic version.

Huggett (1993)

- Endowment economy, no aggregate risk.
- Setup:
  - Discrete time;
  - Continuum of heterogeneous agents;
  - Idiosyncratic endowment risk (labor income stochastic).
- Single bond, a, can be borrowed or saved.
- Borrowling limit, $\underline{a} \leq 0 \leq a_{it}$

Idiosyncratic Markov Income Uncertainty

- Suppose $wl = e$, $F[e'] = \pi(e'|e)$
- Two states: e_l, e_h
- Can be written as

$$V(a,e) = u(c) + \beta \sum_{e'} \pi(e'|e) V(a', e') \tag{16}$$

$$\text{s.t. } c + a' \leq (1+r)a + e \tag{17}$$

$$a' \geq \underline{a} \tag{18}$$

$$c \geq 0, a_0 \text{ given} \tag{19}$$

- Agents want to build precautionary savings again idiosyncratic risk.

Equilibrium

- Define a distribution of agents over assets *as* and endowments e, ψ.
- Stationary equilibrium: aggregate state (ψ) is unchanging.
- Agents move around distribution, but LLN $\rightarrow \psi' = \psi$
- Define $\psi(B)$ such that given transition function P:

$$\psi(B) = \int_S P(x, B) d\psi \tag{20}$$

- $P(x, B)$ the probability that an agent with state x will have state $B \in \beta_S$ next period.
- B is a subset of the state space.

Stationary Equilibrium

- Roughly summarizing Huggett, 1993: A stationary equilibrium for this economy is a tuple (c, a', r, ψ) that satisfy
  1. c and a' solve the workers problem taking prices as given.
  2. Markets clear:
     2.1 consumption = production: $\int c(x) d\psi = \int e d\psi$
     2.2 no net savings: $\int a(x) d\psi = 0$
  3. ψ is stationary:

$$\psi(B) = \int_S P(x, B) d\psi \tag{21}$$

  for all $B \in \beta_S$

Aiyagari (1994)

- Production economy, no aggregate risk.
- Firms employ capital, households save using capital (really assets loaned/borrowed from firm).
- Setup:
  - Discrete time;
  - Continuum of heterogeneous agents;
  - Idiosyncratic hours shocks (labor supply stochastic).
- Capital, k, can be borrowed or saved.
- Borrowling limit, $\underline{k} \leq 0 \leq k_{it}$

Heterogeneous Agent Production Economy

- In a production economy, the agent's problem is given by

$$V(k, \epsilon; \psi) = u(c) + \beta E[V(k'\epsilon'; \psi')] \tag{22}$$
$$\text{s.t. } c + k' \leq (1 + r(K, L) - \delta)k + w(K, L)\epsilon \tag{23}$$
$$k' \geq \underline{k} \tag{24}$$
$$\epsilon \sim \text{ Markov} P(\epsilon'|\epsilon) \tag{25}$$
$$\psi' = \Psi(\psi) \tag{26}$$
$$c \geq 0, k \geq 0, k_0 \text{ given} \tag{27}$$

- ϵ is a markov process that yields hours worked.
- Ψ is an unspecified evolution of the aggregate state (k, ϵ).
- Prices are determined from the firm's problem

Prices - The Firm's Problem

- How we handle prices determines the difficulty of this problem.
- In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} F(K, L) - wL - rK \tag{28}$$

- This yields standard competitive prices for the rental rates.

Information

- What information do workers need in order to be able to solve this problem?
- Current period:
  - interest rate, $r(K, L)$. This is known from being told the aggregates at the beginning of the period.
  - wage rate, $w(K, L)$. This is known from being told the aggregates at the beginning of the period.
- Future:
  - interest rate and wage rate next period.
  - These depend on capital and labor next period.
  - Thus, workers need to predict capital and labor in future.
- Rep. Agent model: just need to know their own decision rule.
- Here: need to know distribution across workers, and their decision rules.

Stationary Recursive Competitive Equilibrium

- A stationary RCE is given by pricing functions r, w, a worker value function $V(k, \epsilon; \psi)$, worker decision rules k', c, a type-distribution $\psi(k, \epsilon)$, and aggregates K and L that satisfy
  1. k' and c are the optimal solutions to the worker's problem given prices.
  2. Prices are formed competitively from the firm's problem.
  3. Consistency between aggregate evolution and individual decision rules: ψ is the stationary distribution implied by worker decision rules.
  4. Aggregates are consistent with individual policy rules: $K = \int k d\psi$, $L = \int \epsilon d\psi$

Return to Capital

- How does return to capital vary by
  - serial corr. (ρ) in labor income (think AR1 process)
  - and CRRA (μ)?

TABLE II

A. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.2$)

$\rho \backslash \mu$	1	3	5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

B. Net return to capital in %/aggregate saving rate in % ($\sigma = 0.4$)

$\rho \backslash \mu$	1	3	5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	−0.3456/37.63

- Higher ρ or μ, more saving, lower return.

Krussell-Smith (1998)

- In the previous model, we relied on the aggregate *certainty* of $\psi(k, \epsilon)$ for a solution by appealing to the law of large numbers.
- i.e., individuals move around the distribution, but those shocks offset and in the aggregate the distribution is unchanged.
- But what happens if there is aggregate *uncertainty*?
- Now the distribution changes in the equilibrium, and we need a way to incorporate this into worker decision rules.
- Krussell-Smith: Aiyagari + aggregate shocks.
- Some details:
  - Idiosyncratic labor shock $\{0,1\}$ markov.
  - Aggregate shocks.
  - Idiosyncratic shock prob. changes with agg. shocks.

Aggregate Uncertainty

- In a production economy, the agent's problem is given by

$$
\begin{aligned}
V(k,\epsilon,z;\psi) &= u(c) + \beta E[V(k'\epsilon',z';\psi')] && (29)\\
\text{s.t. } c + k' &\leq (1 + r(z,K,L) - \delta)k + w(z,K,L)\epsilon && (30)\\
k' &\geq \underline{k} && (31)\\
z' &= \text{ Markov} P(z'|z) && (32)\\
\epsilon &\sim \text{ Markov} P(\epsilon'|\epsilon,z') && (33)\\
\psi' &= \Psi(\psi,z,z') && (34)\\
c \geq 0, k \geq 0, k_0 &\text{ given}, z_0 \text{ given} && (35)
\end{aligned}
$$

- ϵ is a markov process for employment $\epsilon \in \{0,1\}$
- Ψ is an unspecified evolution of the aggregate state.
- z *also* evolves as a markov process.
- Prices are determined from the firm's problem.

Prices - The Firm's Problem

- How we handle prices determines the difficulty of this problem.
- In this economy, a single firm produces using labor (hours) and capital.

$$\Pi = \max_{K,L} zF(K, L) - wL - rK \tag{36}$$

- This yields standard competitive prices for the rental rates.

Laws of Motion

- The future aggregate state enters the probability of employment.
- This means that it impacts **all** of the laws of motion:

$$z' = \text{Markov}P(z'|z) \tag{37}$$

$$\epsilon \sim \text{Markov}P(\epsilon'|\epsilon, z') \tag{38}$$

$$k' \leq (1 + r(z, K, L) - \delta)k + w(z, K, L)\epsilon - c \tag{39}$$

$$\psi' = \Psi(\psi, z, z') \tag{40}$$

- Because shocks to z change employment status and prices.

Recursive Competitive Equilibrium

- An RCE is given by pricing functions r, w, a worker value function $V(k, \epsilon, z; \psi)$, worker decision rules k', c, a type-distribution $\psi(k, \epsilon)$, and aggregates K and L that satisfy
  1. k' and c are the optimal solutions to the worker's problem given prices.
  2. Prices are formed competitively from the firm's problem.
  3. Consistency between aggregate evolution and individual decision rules: ψ is the distribution implied by worker decision rules given the aggregate state.
  4. Aggregates are consistent with individual policy rules: $K = \int k d\psi$, $L = \int \epsilon d\psi$

Type Distribution

- The type distribution is a problem.
- Each policy function and transition depends on the type distribution.
- But the type distribution is time-varying in response to aggregate shocks.
- Alternative: use a smaller number of moments that can be calculated quickly to characterize the type distribution.
- Like a "sufficient statistic" for the type distribution.
- Discuss the solution to this next time.

Business Cycle Effects

- This model is built to handle stochastic shocks.
- How do heterogeneous agents respond over a business cycle?

TABLE 2

AGGREGATE TIME SERIES

Model	Mean(k_t)	Corr(c_t, y_t)	Standard Deviation (i_t)	Corr(y_t, y_{t-4})
Benchmark:				
Complete markets	11.54	.691	.031	.486
Incomplete markets	11.61	.701	.030	.481
$\sigma = 5$:				
Complete markets	11.55	.725	.034	.551
Incomplete markets	12.32	.741	.033	.524
Real business cycle:				
Complete markets	11.56	.639	.027	.342
Incomplete markets	11.58	.669	.027	.339
Stochastic-β:				
Incomplete markets	11.78	.825	.027	.459

Conclusion

- Next time: presentations.