Quantitative Macro-Labor: The Income Fluctuation Problem

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Announcements

- Today: Start heterogeneous agent models.
- First: income fluctuation problem.
- Empirical regularities project/presentation due next week (11/5).
- No need to do both: can turn in just presentation slides.
- But please upload analysis code to cluster or email to me.

Thinking about Uncertainty in Macroeconomic Models

- Uncertainty makes macroeconomic models more difficult to solve.
- We make assumptions about the environment (preferences, technology, etc.) to decrease complexity of problem.
- Euler Equation:

$$u'(c_t) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \underbrace{u'(c_{t+1})}_{Non-linear}] \tag{1}$$

- Each agent chooses consumption and savings based on a
  1. general equilibrium object (given by the decision rules of all other agents)
  2. (potentially highly) non-linear marginal utility.

Thinking about Uncertainty in Macroeconomic Models

- Market clearing:

$$\sum_{i=1}^{N}((1+r_{t+1})a_{i,t+1}+w_{i,t+1}-c_{i,t+1}-a_{i,t+2})=0 \tag{2}$$

- Wealth + Income - (Consumption + Savings) = 0
- Now we have to find decision rules that satisfy

$$u'(c_{i,t})=\beta E[(1+r_{t+1})u'(c_{i,t+1})] \tag{3}$$

- Imposing decision rules as a function of worker state ($\hat{S}_{i,t}$):

$$\sum_{i=1}^{N}((1+r_{t+1})a_{i,t+1}(\hat{S}_{i,t+1})+w_{i,t+1}(\hat{S}_{i,t+1})) \tag{4}$$

$$-\sum_{i=1}^{N}(c_{i,t+1}(\hat{S}_{i,t+1})-a_{i,t+2}(\hat{S}_{i,t+2}))=0 \tag{5}$$

Thinking about Uncertainty in Macroeconomic Models

- Typical assumptions in macroeconomics are a convex combination of
  1. certainty equivalence:
$$u'(\bar{c}_{i,t}) = \beta E[(1 + \underbrace{r_{t+1}}_{GE}) \underbrace{u'(\bar{c}_{i,t+1})}_{Closer\ to\ Linear}] \tag{6}$$
  2. linearized decision rules:
$$\sum_{i=1}^{N}((1 + r_{t+1})a_{i,t+1} + w_{i,t+1} - c_{i,t+1} - a_{i,t+2}) = 0 \tag{7}$$
$$\sum_{i=1}^{N}((1 + r_{t+1})\beta_a\hat{S}_{i,t+1} + \beta_w(\hat{S}_{i,t+1}) - \beta_c\hat{S}_{i,t+1} - \beta_a\hat{S}_{i,t+2}) = 0 \tag{8}$$
- Can be expressed as matrix & solved quickly on computer.

So far

- We've thought about worlds in which some markets are imperfect:
  1. labor market frictions: information is absent, there is a time/monetary cost associated with obtaining it.
  2. risk-neutral preferences: workers still have access to some type of complete markets.
- Today: a different route. Workers cannot insure against income uncertainty.
- Explore using different preferences:
  1. Certainty Equivalence - Quadratic Utility.
  2. Constant Absolute Risk Aversion - Exponential Utility.
  3. Constant Relative Risk Aversion.
- These each imply different ways in which agents respond to income shocks and uncertainty.

Risk

- How do we typically think about risk in economic models?
- Absolute Risk Aversion:

$$AR = -\frac{u''(c)}{u'(c)} \tag{9}$$

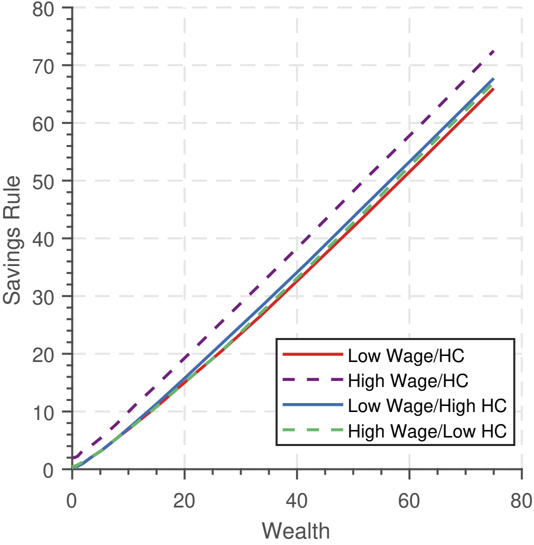
- A measure of the agent's preference for smoothing utility regardless of their wealth.
- Relative Risk Aversion:

$$RRA = -\frac{u''(c)c}{u'(c)} \tag{10}$$

- Preference for smoothing utility relative to their level of wealth.
- Probably most reasonable are "DARA" "CRRA"
- These will have different implications for savings and consumption.

When approximations work

- Last time, we talked about interpolation.
- Linear interpolation is “cheap” and works *for most of the distribution*:



Introduction

- In the case of quadratic utility, we will see that agents don't change their consumption choices when faced with shocks.
- Uncertainty still decreases expected utility (levels), but does not change choices (marginal utility).
- Why is this relevant? One solution technique (LQ) assumes that agents have a quadratic utility function (locally risk-neutral).
- We will see that this is sometimes not a great assumption.

Quadratic Utility

- Utility is given by the following:

$$\max E[\sum_{t=0}^{\infty} \beta^t (aC_t - bC_t^2)] \tag{11}$$

$$\begin{aligned} \text{s.t. } A_{t+1} &= (1+r)A_t + Y_t - C_t && (12) \\ Y_{t+1} &= \rho Y_t + \epsilon_{t+1} && (13) \end{aligned}$$

Euler Equation

- Do the usual steps to find the Euler Equation:

$$V(A) = \max_{C,A'} aC_t - bC_t^2 + \beta E[V(A')] \tag{14}$$

$$\text{s.t. } A' = (1+r)A + Y - C \tag{15}$$
$$Y' = \rho Y + \epsilon' \tag{16}$$

$$\frac{\partial V}{\partial C} = a - 2bC - \lambda \tag{17}$$
$$\frac{\partial V}{\partial A'} = -\lambda + \beta E[\frac{\partial V}{\partial A'}] \tag{18}$$
$$\frac{\partial V}{\partial A} = (1+r)\lambda \tag{19}$$
$$\Rightarrow C = \beta(1+r)E[C'] \tag{20}$$

Certainty Equivalence

- Assume that $\beta = \frac{1}{1+r}$:

$$\Rightarrow C = E[C'] \tag{21}$$

- Now suppose there are two states of the world: high and low.

$$C = P_h C_h + P_l C_l \tag{22}$$

- Euler Equation (21) $\Rightarrow$ gamble yields same choices as:

$$C = C_m \tag{23}$$

- i.e., workers make savings decisions *as though they are receiving the average consumption with certainty*.

Prudence

- Agents in this economy are not "prudential."
- That is, they don't change their choices based upon uncertainty about the future.
- Another way to express this is in the third derivative of the utility function:
$$U''' = 0 \tag{24}$$
- This captures the response of marginal utility (i.e., decisions) to uncertainty.
- Marginal utility changes linearly, so any convex combination is equal to the expected value.
- i.e., a gamble does not change expected marginal utility.

Random Walk

- Can show for the AR(1) case:

$$C_t - C_{t-1} = \frac{r}{1 + r - \rho}\epsilon \tag{25}$$

- Now, consider the case in which income shocks are iid:

$$Y_{t+1} = Y_t + \epsilon_{t+1} \tag{26}$$

- Then the difference in consumption becomes:

$$C_t - C_{t-1} = \epsilon_t \tag{27}$$

- In other words, the agent consumes all of the shock in each period (will also happen with CRRA and autarky).
- Why? Equalizing expected consumption over time.

Takeaways from Certainty Equivalence

- In the quadratic utility world, uncertainty does not change an agents decision when compared with an identical income stream.
- In the case of CARA utility, we will see that agents have precautionary savings that result from curvature in the utility function.
- The choices are the same as they would be under complete markets.
- When might this be appropriate? When decisions are nearly linear (i.e., super wealthy).

Introduction to Prudence

- Now, use CARA (Constant Absolute Risk Aversion) preferences to think about world in which certainty equivalence does not hold.
- Now, we will allow agents to be prudential in their savings response to future uncertainty.

Constant Absolute Risk Aversion Utility

- The maximization problem is given by

$$\max E[\sum_{t=0}^{\infty} -\frac{1}{\alpha}\exp(-\alpha C_t)] \tag{28}$$

$$\begin{aligned} \text{s.t. } A_{t+1} &= A_t + Y_t - C_t && (29) \\ Y_t &= Y_{t_1} + \epsilon_t, \epsilon_t \sim N(0, \sigma^2) && (30) \end{aligned}$$

- Assumptions: unit root ($\rho = 1$), $r = 0$, $\beta = \frac{1}{1+r}$
- Key difference: first derivative (i.e., policy functions), no longer linear.

Euler Equation

- Bellman Equation (implicitly assume $\beta = \frac{1}{1+r}$):

$$V(A) = \max_{C,A'} -(\frac{1}{\alpha})\exp(-\alpha C) + E[V(A')] \tag{31}$$

$$\begin{aligned} \text{s.t. } A' &= A + Y - C && (32) \\ Y' &= Y + \epsilon' && (33) \end{aligned}$$

$$\frac{\partial V}{\partial C} = \exp(-\alpha C) - \lambda \tag{34}$$

$$\frac{\partial V}{\partial A'} = -\lambda + E[\frac{\partial V}{\partial A'}] \tag{35}$$

$$\frac{\partial V}{\partial A} = \lambda \tag{36}$$

$$\Rightarrow \exp(-\alpha C) = E[\exp(-\alpha C')] \tag{37}$$

Euler Equation

- Bellman Equation (implicitly assume $\beta = \frac{1}{1+r}$):

$$\exp(-\alpha C) = E[\exp(-\alpha C')] \tag{38}$$

- For normally distributed random variables, the following holds:

$$E[exp(x)] = exp(E[x] + \sigma_x^2/2) \tag{39}$$

- Thus, we can rewrite the Euler Equation as

$$\exp(-\alpha C) = E(exp(-\alpha C' + \alpha^2 \sigma^2/2)) \tag{40}$$

$$\Rightarrow C' = C + \frac{\alpha\sigma^2}{2} + \nu \tag{41}$$

- ν expectation error.

Policy Function

- Policy function:

$$\Rightarrow C' = C + \frac{\alpha\sigma^2}{2} + \nu \tag{42}$$

- This says that consumption is *increasing* ex-ante in response to uncertainty, measured by σ^2.
- What does this mean about life-cycle consumption?
- We would expect it to be upward-sloping, at least initially.

Consumption in time t

- In finite life-cycle model (consumption unbounded in infinite horizon), exit model at time T.
- Can show:

$$C_t = (\frac{1}{T-t})A_t + Y_t - \frac{\alpha(T-t-1)\sigma^2}{4} \tag{43}$$

- Certainty equivalence: last term is equal to zero. i.e., cake-eating problem.
- Agents consume less than they would if their income stream was certain!

Prudence

- What is different in this case?
- Agents are prudential: $U''' > 0$.
- The Euler Equation is given by:

$$\exp(-\alpha C) = E[\exp(-\alpha C')] \tag{44}$$

- Suppose $C = C'$, then consider Jensen's Inequality:

$$f(E(C)) < E[f(C)] \tag{45}$$

- For Euler Equation to hold in equilibrium, $C \downarrow$, i.e. must increase current marginal utility and reduce future marginal utility.
- Agents save in excess of what they would under certainty!

CARA Utility

- When CARA agents cannot perfectly insure, they change their choices from the certainty equivalence (quadratic utility) case.
- Unfortunately, CARA has some problems: Marginal utility is finite when consumption is equal to zero.
- CRRA utility will solve this problem, but is more challenging to solve.

CRRA Preferences

- Now, we will start to think about an economy in which agents have **C**onstant **R**elative **R**isk **A**verse preferences.
- i.e., power utility.
- What else does this mean? Key difference:
- Agents are very unhappy when they starve:

$$u'(0) = \infty \tag{46}$$

- Seems like a reasonable assumption.
- Cover this in heterogeneous agent models next time.

Next time

- First wave of heterogeneous agent models: how do aggregates change when *individual idiosyncratic* uncertainty is uninsurable.
- In other words: when agents must accumulate *precautionary savings* to insure against income shocks.
- Key "first wave" papers (no particular order):
  - Huggett (1993): Incomplete markets exchange economy with GE interest rate.
  - Imrohoroglu (1989): Individual and aggregate uncertainty with fixed interest rate.
  - Aiyagari (1994): Incomplete markets production economy with GE interest rate.
  - Bewley (1986): Individual uncertainty with fixed interest rate.
- Empirical regularities project due next Tuesday.
- Presentations next week (11/5)