Quantitative Macro-Labor: General Equilibrium Search and Matching

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Announcements

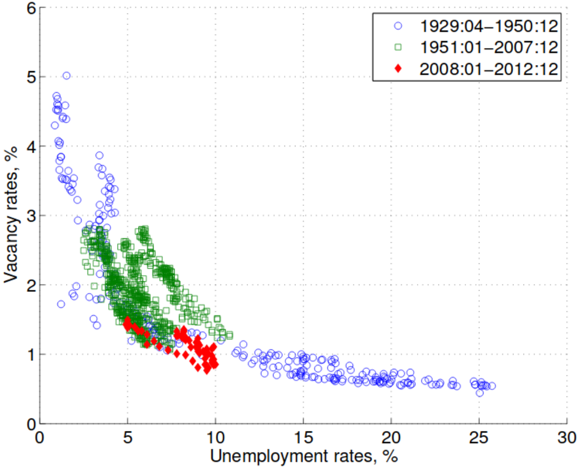
- Today: the Mortensen and Pissarides model (canonical equilibrium search)
- Everyone should have started the data project.
- Next Tuesday: Research proposal and presentations.

Arrival Rates of Job Offers

- So far, we have assumed that the arrival rate of job offers is *exogenous*: regardless of equilibrium, the frequency with which you receive an offer is the same.
- Consider an example:
  1. There is a productivity downturn:
  2. How does a firm respond?
  3. BM, BC, PVR: the quality of the offer distribution deteriorates, but searchers receive offers at the same rate.
- Essentially, slackness in the labor market is due to worker selectivity, not due to decisions made by the firm.
- Obviously, firms do respond.

The Beveridge Curve

- Another implication: there is no relationship between unemployment and vacancy creation.



(source: Petrosky-Nadeau and Zhang, 2017)

Hazard vs. Arrival Rate

- Unmatched firm problem in Burdett-Mortensen:

$$\pi = \max_{w} (p - w) l(w | w_R, F)$$

- The hazard rate is an equilibrium object; the arrival rate is not.
- But what if firms can adjust along the extensive margin in addition to the intensive margin?
- "Vacant" firm's problem:

$$\pi^V(w) = -\kappa + q(.) J(w)$$

- q is the worker-finding rate. Now an equilibrium object.

The DMP Model ("Ch. 1 of Pissarides (2000)")

- Agents:
  1. Employed workers;
  2. unemployed workers;
  3. vacant firms;
  4. matched firms.
- Linear utility ($u = b, u = w$) and production $y = p > b$.
- Matching function:
  1. Determines *number* of meetings between firms & workers.
  2. Args: levels searchers & vacancies ($U = u \times L, V = v \times L$)
  3. Constant returns to scale (L is lab. force):

$$M(uL, vL) = uL \times M(1, \frac{v}{u}) = uL \times p(\theta)$$

  4. where $\theta = \frac{v}{u}$ is "labor market tightness"
  5. Match rates:

$$\underbrace{p(\theta)}_{Worker} = \theta \underbrace{q(\theta)}_{Firm}$$

Worker Value Functions

- Value functions:
  1. Employed at wage w: $W(w)$
  2. Unemployed: U.
- Unemployed flow value:

$$rU = b + p(\theta)E[W(w) - U]$$

- Employed flow value:

$$rW(w) = w + \delta[U - W(w)]$$

Firm Value Functions

- Value functions:
  1. Filled, paying wage w: $J(w)$
  2. Vacant V.
- Vacant flow value:

$$rV = -\kappa + q(\theta)E[J(w) - V]$$

- Matched flow value:

$$rJ(w) = (p - w) + \delta[V - J(w)]$$

- Free entry equilibrium condition:

$$rV = 0$$
$$\rightarrow \frac{\kappa}{E[J(w)]} = q(\theta)$$

- This is just a market clearing condition!

Equilibrium Objects

- Three key equilibrium objects:
  1. Wages;
  2. unemployment;
  3. $\theta = \frac{v}{u}$ (vacancies).
- How we determine each of these is largely a modeling decision.
- Steady-state: pin down unemployment via flow equation.
- Free-entry: Assume that firms always post vacancies so that free entry binds.
- Wages: Assume that wages are determined by a surplus- (profit) sharing rule.

Steady-State Unemployment

- Flow of unemployment:

$$\dot{u} = \delta(1-u) - p(\theta)u$$

- Steady-state:

$$\begin{aligned} 0 &= \delta(1-u) - p(\theta)u \\ p(\theta)u &= \delta(1-u) \\ u &= \frac{\delta}{\delta + p(\theta)} \end{aligned}$$

- Same as McCall with $\alpha = p(\theta)$.
- (Note: no heterogeneity & $p > b \rightarrow$ all wages accepted.)

Free Entry

- Free entry $V = 0$:

$$rJ(w) = (p - w) + \delta[\cancel{V} - J(w)]$$
$$(r + \delta)J(w) = (p - w)$$

- Vacancy creation condition (i.e., free entry imposed):

$$q(\theta) = \frac{\kappa}{E[J(w)]}$$
$$q(\theta) = \frac{\kappa(r + \delta)}{(p - w)}$$
$$\theta = q^{-1}\left(\frac{\kappa(r + \delta)}{(p - w)}\right)$$

- Thus, mapping between wages and θ. 1 equation, 2 unknowns.
- Need equation to determine wages in equilibrium.

Wage Determination

- Previous wage determination approaches:
  1. Burdett-Mortensen: wage-posting game, contracts non-negotiable.
  2. Postel-Vinay & Robin: wage-posting game, contracts set wages to equalize value of outside offer.
- Here: workers and firms bargain over the surplus of a match.
- Surplus of a match:

$$S(w) = W(w) + J(w) - U - \cancel{V}$$
$$S(w) = W(w) + J(w) - U$$

- Nash Bargaining splits this surplus according to a bargaining weight, β:

$$w = argmax_w \underbrace{(W(w) - U)^{\beta}}_{Net\ Utility} \underbrace{(J(w) - V)^{1-\beta}}_{Net\ Profits}$$

Wage Determination

- Nash Bargaining splits this surplus according to a bargaining weight, β:

$$w = argmax_w \underbrace{(W(w) - U)^\beta}_{Net\ Utility} \underbrace{(J(w) - V)^{1-\beta}}_{Net\ Profits}$$

$$0 = \beta(W(w) - U)^{\beta-1}(J(w) - V)^{1-\beta}\frac{\partial W}{\partial w} + (1-\beta)(J(w) - V)^{-\beta}(W(w) - U)\frac{\partial J}{\partial w}$$

- $\frac{\partial W}{\partial w} = 1$, $\frac{\partial J}{\partial w} = -1$:

$$\beta(\frac{J(w)}{W(w) - U})^{1-\beta} = (1-\beta)(\frac{W(w) - U}{J(w)})^\beta$$

$$\beta(J(w) + W(w) - U) = W(w) - U$$

$$\beta S(w) = W(w) - U$$

Wage Determination

- Nash Bargaining splits this surplus according to a bargaining weight, β:

$$w = argmax_w \underbrace{(W(w) - U)^{\beta}}_{Net\ Utility} \underbrace{(J(w) - V)^{1-\beta}}_{Net\ Profits}$$

$$w \text{ solves } (W(w) - U) = \beta(W(w) + J(w) - U) = \beta S(w)$$

- Plug in for each of these:

$$\begin{aligned}
(1-\beta)[W(w) - U] &= \beta J(w) \\
\beta J(w) &= (1-\beta)[w - \delta(U - V(w)) \\
&\quad - b - p(\theta)(W(w) - U)] \\
(1-\beta)(w-b) &= \beta J(w) + (1-\beta)(p(\theta)+\delta)[W(w) - U] \\
(1-\beta)(w-b) &= \beta(p - w - \delta J(w)) \\
&\quad + (1-\beta)(p(\theta)+\delta)[W(w) - U]
\end{aligned}$$

Wage Determination

- Note that $\beta S(w) = [W(w) - U]$

$$(1-\beta)(w-b) = \beta(p - w - \delta J(w)) \\ + (1-\beta)(p(\theta)+\delta)\beta S(w)$$

- And $(1-\beta)S(w) = J(w) \rightarrow S(w) = \frac{J(w)}{1-\beta}$

$$(1-\beta)(w-b) = \beta(p - w - \delta J(w)) \\ + (1-\beta)(p(\theta)+\delta)\beta\frac{J(w)}{1-\beta} \\ w = (1-\beta)b + \beta p + p(\theta)\beta J(w)$$

- Free entry condition: $q(\theta) = \frac{\kappa}{J(w)} \rightarrow p(\theta) = \frac{\theta\kappa}{J(w)}$

$$w = (1-\beta)b + \beta p + \beta\theta\kappa$$

Computation

- How would we solve this model?
- Need way to compute three equilibrium objects:
  1. Wages;
  2. unemployment;
  3. $\theta = \frac{v}{u}$ (vacancies).
- How we determine each of these is largely a modeling decision.
- Steady-state: pin down unemployment via flow equation.
- Free-entry: Assume that firms always post vacancies so that free entry binds.
- Wages: Assume that wages are determined by a surplus- (profit) sharing rule.
- Computation:
  - Wages, vacancies: depend on surplus.
  - Unemployment: law of motion.
- Here: add aggregate shocks.

Worker Value Functions

- Value functions:
  1. Employed at wage w: $W(w)$
  2. Unemployed: U.
- Unemployed flow value:

$$rU(z) = b + p(\theta)E[W(w,z) - U(z)] + \gamma E[U(z') - U(z)]$$

- Employed flow value:

$$\begin{aligned} rW(w,z) &= w(z) + \delta[U(z) - W(w,z)] \\ &\quad + \gamma E[W(w',z') - W(w,z)] \end{aligned}$$

Firm Value Functions

- Value functions:
  1. Filled, paying wage w: $J(w)$
  2. Vacant V.
- Vacant flow value:

$$rV(z) = -\kappa + q(\theta(z))E[J(w,z) - V(z)] + \gamma[V(z') - V(w,z)]$$

- Matched flow value:

$$\begin{aligned} rJ(w,z) = (z + p - w) + \delta[V(z) - J(w,z)] \\ + \gamma[J(w',z') - J(w,z)] \end{aligned}$$

- Free entry equilibrium condition:

$$\begin{aligned} rV = 0 \\ \rightarrow \frac{\kappa}{E[J(w,z)]} = q(\theta) \end{aligned}$$

Computation

- Surplus of a match:

$$S(w,z) = W(w,z) + J(w,z) - U(z) - \cancel{V(z)}$$
$$S(w,z) = W(w,z) + J(w,z) - U(z)$$

- Plugging in and using $\beta S(w,z)$ is workers surplus and $(1-\beta)S(w,z)$ is firm surplus:

$$S(z) = \frac{p+z}{r+\delta+\gamma} - \frac{b+\theta\kappa\frac{\beta}{1-\beta}}{r+\delta+\gamma} + \frac{\gamma}{r+\delta+\gamma}\int_{z'} S(x)dF(x)$$

- This is just a contraction: $\frac{\gamma}{r+\delta+\gamma} < 1$.
- Pick $S_0(z_i) = 0, \ \forall \ i$ and iterate.
- Yields vacancies $q(\theta) = \frac{\kappa}{(1-\beta)S(z)}$ and wages $(w = \beta S(z))$.
- Solve discrete time version for homework due Thursday after next (10/6).
- Then simulate unemployment dynamics for 1000 periods.

Conclusion

- Next Tuesday: Research proposal and presentations.
- Thursday: Endogenous separation? **Hosios Condition**? Directed Search?
- Start your data projects.