

# Quantitative Macro-Labor: Disciplining Wage Contracts with Convex Preferences

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# Announcements

- ▶ Today: Extend Burdett-Mortensen to environment where firms post *wage-tenure contracts* instead of wages.
- ▶ Research Proposal/Introduction: Due next Thursday.
- ▶ Short presentations (probably) the following week.
- ▶ May instead start section on Market Power.

# The Burdett-Mortensen Model

- ▶ What is an (one of many) important and realistic feature of the labor market missing in the standard McCall model?
- ▶ The ability to search while employed.
- ▶ Some statistics:
  1. 50% of all hires are job-to-job hires (Census)
  2. Movement up job ladder accounts for 50% of wage growth for young workers (Topel and Ward, 1992)
  3. 70% of fall in hires during Great Recession was J2J.
- ▶ But, wages also increase with *tenure*.
- ▶ BM model can't account for this.

# The Burdett-Mortensen OTJS Model

▶ Basic idea:

1. Workers can be in one of two states: employed or unemployed, with value functions  $V, U$ .
2. Firms post wages, i.e., a given distribution of wages,  $w \in [\underline{w}, \bar{w}]$ ,  $w \sim F(\cdot)$ .
3. Unemployed receive job offers at exogenous rate  $\alpha$ , no prior info.
4. Employed job offers at exogenous rate  $\lambda$ , no prior info.
5. Separate two ways: exogenously (rate  $\delta$ ) and via thru OTJS (rate  $\lambda[1 - F(w)]$ )
6. Linear utility:  $u(c) = b$  or  $u(c) = w$ .

▶ **Firms cannot respond to outside offers.**

▶ Are these contracts optimal?

# The Burdett-Mortensen Model

- ▶ Flow value of unemployment:

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V(x) - U, 0\} dF(x) \quad (1)$$

- ▶ Employment:

$$rV(w) = w + \lambda \int_{\underline{w}}^{\bar{w}} \max\{V(x) - V(w), 0\} dF(x) + \delta(U - V(w)) \quad (2)$$

- ▶ Thus, the reservation wage is

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} \frac{[1 - F(x)]}{r + \delta + \lambda[1 - F(w_R)]} dx \quad (3)$$

# The Firm

- ▶ Assume equilibrium conditions &  $\lambda = \alpha$ .
- ▶ Define  $\pi^V$  as the profits of a vacant firm.
- ▶ Firm profit function:

$$\pi^V = \max_w (p - w)l(w|w_R, F) \quad (4)$$

$$\pi^V(w|w_R, F) = (p - w)l(w|w_R, F) \quad (5)$$

$$l(w|w_R, F) = \frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2} \quad (6)$$

- ▶ What is  $l$ ? It is the probability of meeting a workers *and* the expected duration a worker employed at wage  $w$  will be employed with a firm.

## Matched Firm Value

- ▶ Matched firm profit function:

$$r\pi^F(w|w_R, F) = (p - w) + \alpha[1 - F(w)](\pi^V - \pi^F(w|w_R, F)) \\ + \delta(\pi^V - \pi^F(w|w_R, F)) \quad (7)$$

(8)

- ▶ Would a matched firm prefer to retain a worker?

$$r\pi^F(w_R|w_R, F) = (p - w_R) + \alpha[1 - \cancel{F(w_R)}](\pi^V - \pi^F(w_R|w_R, F)) \\ + \delta(\pi^V - \pi^F(w_R|w_R, F)) \quad (9)$$

$$r\pi^F(w_R|w_R, F) = (p - w_R) + (\alpha + \delta)(\pi^V - \pi^F(w_R|w_R, F)) \quad (10)$$

- ▶  $\pi^F \geq \pi^V$ . Why is this?
- ▶ The firm would prefer to retain a worker.
- ▶ Maybe there is room for better contracts?

## Burdett and Coles (2003)

- ▶ Another way of thinking about on-the-job search: moral hazard.
- ▶ Firm can't contract on worker behavior: worker can leave anytime it finds a higher wage.
- ▶ How could we allow a firm to handle this?
  1. Respond to outside offers. (doesn't make sense in model with identical productivity).
  2. Write contracts that change wages to mitigate moral hazard.
- ▶ What does option 2 mean? They will increase wages the longer a worker remains employed.
- ▶ Backloaded contracts: option value larger so worker wants to stay.



# Outline (Burdett-Coles, 2003)

- ▶ Preferences and Technology:
  1. Workers can be in one of two states: employed or unemployed, with value functions  $V_E, V_U$ .
  2. Firms post wage-contracts: distribution of *promised values*,  $V \sim F(\cdot)$ .
  3. Workers receive job offers at exogenous rate  $\lambda$ .
  4. Separate two ways: “exit the model” (rate  $\delta$ ) and via thru OTJS (rate  $\lambda[1 - F(V)]$ )
  5. Risk-aversion:  $u(c) = u(b)$  or  $u(c) = u(w)$ .
  6. Rate of time preference:  $r = 0$  (simplification for analytical results).
- ▶ **Firms cannot respond to outside offers.**
- ▶ Instead, they increase wages over time to ensure that a worker does not leave early.

# Contracts

- ▶ Rather than wages, a firm offers a promised value, and can distribute it in any way over the life of the contract.
- ▶ Optimally, they want to backload the contract.
- ▶ Why? Because workers are less likely to leave if continuation value is high.
- ▶ Wage-tenure contracts:
  1. Wage  $w(\cdot) \geq 0$ .
  2. Where tenure,  $t$ , is its only argument.
  3. By assumption, equilibrium is symmetric, i.e., all contracts are the same given  $t$ .

## Worker Value Functions

- ▶ Unemployed flow value (bc  $r = 0$ ):

$$\delta V_U = u(b) + \lambda \int_{V_U}^{\bar{V}} [x - V_U] dF(x) \quad (11)$$

- ▶ Employed value function

$$\begin{aligned} \delta V_E(t|w(.)) - \frac{dV_E(t|w(.))}{dt} = & u(w(t)) \\ & + \lambda \int_{V_E(t|w(.))}^{\bar{V}} [V_x - V_E(t|w(.))] dF(V_x) \end{aligned} \quad (12)$$

- ▶ Currently, optimal contract unspecified.
- ▶ What is  $\frac{dV_E(t|w(.))}{dt}$ ?

## Firm Value Functions

- ▶ Define a worker survival probability (i.e., doesn't leave or die.) through tenure  $t$ .

$$\psi(t|w(.)) = e^{-\int_0^t [\delta + \lambda(1 - F(V(s|w(.))))] ds} \quad (13)$$

- ▶ Define  $G(V)$  the steady state number of workers with lifetime utility less than  $V$ .
- ▶ Then,

$$\Omega = \underbrace{[\lambda G(V_0)]}_{\text{Finding Rate}} \int_0^\infty \underbrace{\psi(t|w(.))}_{\text{Survival}} \underbrace{[p - w(t)]}_{\text{Flow profits}} dt \quad (14)$$

- ▶ Where  $\Omega$  is the equilibrium profits of a firm posting contract  $w(.)$  s.t.,  $E[u(w(.))] = V_0$ .

# Optimal Contract

- ▶ Fundamental question: Why do we write contracts?
- ▶ Firm dynamic programming problem

$$\max_{w(\cdot) \geq 0} \int_0^{\infty} \psi(t|w(\cdot)) [p - w(t)] dt \quad (15)$$

- ▶ What are the two terms here?

$$\max_{w(\cdot) \geq 0} \int_0^{\infty} \underbrace{(\psi(t|w(\cdot)))p}_{\text{Gains}} - \underbrace{\psi(t|w(\cdot))w(t)}_{\text{Losses}} dt \quad (16)$$

- ▶ Subject to  $V(0|w(\cdot)) = V_0$ .
- ▶ “Free Entry”:

$$\Omega^*(V_0) = \lambda G(V_0) \pi^*(0|V_0) \quad (17)$$

# Equilibrium

► Assumptions:

1. Convex preferences:  $u_c > 0$ ,  $u'' < 0$  and exists, and  $\lim_{c \rightarrow 0} u(c) = -\infty$
2.  $\forall V_0 \in (\underline{V}, \bar{V})$ ,  $F$  is continuously diff'ble &  $F'(V_0) > 0$ .

► From the paper:

*A market equilibrium is:*

1. a distribution of starting payoffs  $F$ .
2. Optimal wage-tenure contracts  $w^*(\cdot | V_0)$ ,  $V_0 \geq V_U$ .
3. Optimal stopping-time solutions to both worker problems.
4. Worker value distribution  $G$  consistent with worker flows.
5. Optimal posting game:

$$\Omega^*(V_0) = \bar{\Omega} \forall V_0 \in [\underline{V}, \bar{V}] \quad (18)$$

$$\Omega^*(V_0) \leq \bar{\Omega} \text{ otherwise} \quad (19)$$

## Optimal Contract

- ▶ Fundamental question: Why do we write contracts?
- ▶ Firm dynamic programming problem

$$\max_{w(\cdot) \geq 0} \int_0^{\infty} \psi(t|w(\cdot)) [p - w(t)] dt \quad (20)$$

$$\text{s.t. } w(\cdot) \geq 0 \quad (21)$$

$$V(0|w(\cdot)) = V_0 \quad (22)$$

- ▶ Continuous-time optimization (Hamiltonian) where

$$\dot{\psi} = -[\delta + \lambda(1 - F(V))] \psi \quad (23)$$

$$\dot{V} = \delta V - u(w) - \lambda \int_V^{\bar{V}} [x - V] dF(x) \quad (24)$$

- ▶ Hamiltonian:

$$H = \psi[p - w] + x_{\psi} \dot{\psi} + x_V \dot{V} \quad (25)$$

- ▶ Optimal path of the wage would solve this problem.

## Optimal Contract

- ▶ Solving the contracting problem yields the following contract:

$$\frac{u'(w^*(0|\cdot))}{u'(w^*(\tau|\cdot))} = 1 + u'(w^*(0|\cdot)) \int_0^\tau \lambda F'(V^*(t|\cdot)) \pi^*(t|\cdot) dt \quad (26)$$

- ▶ Strictly increasing and concave utility:

$$\frac{u'(w^*(0|\cdot))}{u'(w^*(\tau|\cdot))} > 1 \rightarrow w^*(\tau) > w^*(0) \quad (27)$$

- ▶ Increasing wage at time  $\tau$  decreases quit rate over  $[0, \tau]$ :

$$\underbrace{u'(w^*(0|\cdot))}_{\text{Slope}} \int_0^\tau \underbrace{\lambda F'(V^*(t|\cdot))}_{\Delta \text{ Quit Rate}} \underbrace{\pi^*(t|\cdot)}_{\text{PDV of Profits}} dt \quad (28)$$

- ▶ Moral hazard: reward those who do not quit.



## Market Equilibrium

- ▶ Similar to the BM model (but with more algebra), can show that there exists a unique equilibrium, with the following:

$$\left(\frac{\delta}{\lambda + \delta}\right)^2 = \frac{p - \bar{w}}{p - \underline{w}} \quad (29)$$

$$u(\underline{w}) = u(b) - \frac{\sqrt{p - \underline{w}}}{2} \int_{\underline{w}}^{\bar{w}} \frac{u'(x) dx}{\sqrt{p - x}} \quad (30)$$

- ▶ With a wage-tenure contract given by

$$\frac{dw}{dt} = \frac{\delta}{\sqrt{p - \bar{w}}} \frac{p - w}{u'(w)} \int_w^{\bar{w}} \frac{u'(x) dx}{\sqrt{p - x}} \quad (31)$$

# Market Equilibrium

- ▶ Pins down initial wage & thus contract. (Burdett and Coles, 2003)

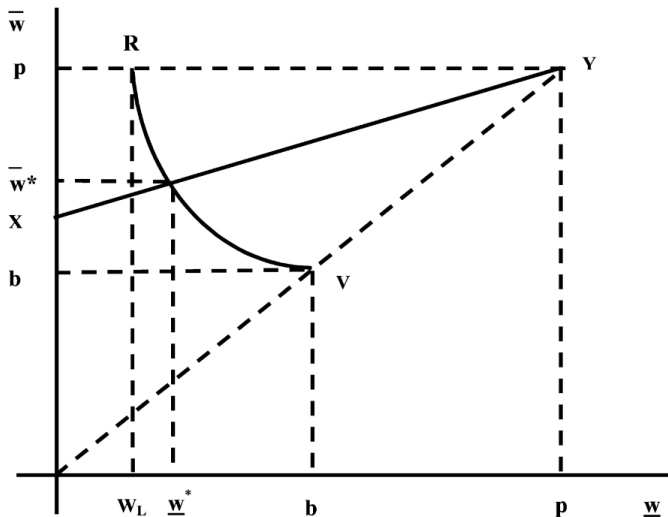


FIGURE 1.

## Risk Aversion

- ▶ We breezed through preferences earlier.
- ▶ What role does consumption risk play here?
  1. Way to give workers a preference over when they receive consumption.
  2. Opportunity for firm to provide insurance against shocks.
- ▶ They use different CRRA thresholds: (Burdett and Coles, 2003)

TABLE I

$\sigma$	$\underline{w}$	$\bar{w}$	$F_w(\underline{w})$
0.0	0.600	4.964	1.000
0.1	1.333	4.970	0.911
0.2	1.751	4.973	0.864
0.4	2.223	4.977	0.803
0.8	2.728	4.981	0.727
1.4	3.126	4.985	0.657
$\infty$	4.600	4.997	0.000

## Consumption Risk

- ▶ What is strange about this table? (Burdett and Coles, 2003)

TABLE I

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$\infty$	4.600	4.997	0.000

- ▶ There was no precautionary savings here.
- ▶ i.e., firms attracted workers by offering *insurance* against low consumption via a guaranteed contract.
- ▶ But, transitory consumption risk already mitigated by autarky and UI.

## Next Time

- ▶ Postel-Vinay and Robin (2002): Sequential auctions (i.e., firm *can* respond to outside offers)
- ▶ Next Thursday, introduction/research proposal due.
- ▶ Presentations the following Tuesday (probably).
- ▶ Start your data projects soon!