Quantitative Macro-Labor: Extending the McCall Model with On-the-Job Search

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Announcements

- Today: On-the-Job Search (Burdett-Mortensen, 1998)
- You should have logged into the cluster/storage by now.
- Set up your own folder under "student_folders". Use the format first initial-last name (i.e., bgriffy).
- Never put spaces in your directories (use underscores _)

Running Code on the Cluster I

- There is code to generate the following datasets:
  1. PSID: you'll need Python and Stata, and run "PSIDMaster.py" after choosing variables.
  2. SIPP: you'll need Stata, run "SIPPMaster.do"
  3. NLSY: pick variables from NLSY website.
- Run "Bewley1986" Matlab code as a test on cluster
- Please install the anaconda distribution of Python.
- Get Matlab from the UAlbany software system.

Running Code on the Cluster II

- Open terminal where your “slurm-file” is located.
- Type “sbatch slurmFileName” and it will run.
- Change the email in the slurm file so that I don’t get a bunch of emails.
- Run “Bewley1986” Matlab code as a test.

Why are Similar Workers Paid Differently?

- Posed by Dale Mortensen in his book "Wage Dispersion"
- Abowd, Kramarz, and Margolis (1999): "That... observably equivalent individuals earn markedly different compensation and have markedly different employment histories–is one of the enduring features of empirical analyses of labor markets..."
- What are some possible reasons?
  1. Ability
  2. Selectivity
- McCall model: after we control for ability (or whatever), worker selectivity generates all wage dispersion.
- The more selective workers are, the smaller the dispersion is.
- Hornstein, Krusell, Violante: Simple models can't match dispersion.

McCall Model Review

- Model equilibrium characterized by two flow equations and a policy function: unemployment, employment and reservation wage.
- Unemployment:

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V - U, 0\} dF(w) \qquad (1)$$

- Employment:

$$rV(w) = w - \delta(V(w) - U) \qquad (2)$$

- Reservation strategy:

$$w_R = b + \frac{\alpha}{r+\delta} \int_{w_R}^{\bar{w}} [1 - F(w)] dw \qquad (3)$$

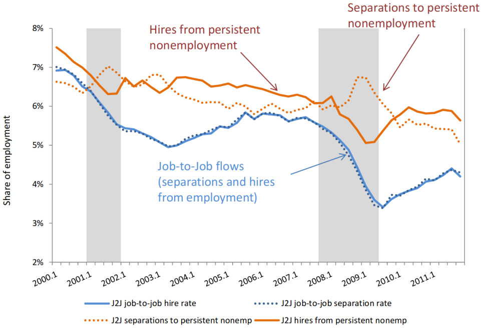
$$w_R = b + \frac{\alpha(1 - F(w_R))}{r+\delta} \int_{w_R}^{\bar{w}} (w - w_R) \frac{dF(w)}{1 - F(w_R)} \qquad (4)$$

The Burdett-Mortensen Model

- What is an (one of many) important and realistic feature of the labor market missing in the standard McCall model?
- The ability to search while employed.
- Some statistics:
  1. 50% of all hires are job-to-job hires (Census)
  2. Movement up job ladder accounts for 50% of wage growth for young workers (Topel and Ward, 1992)
  3. 70% of fall in hires during Great Recession was J2J.

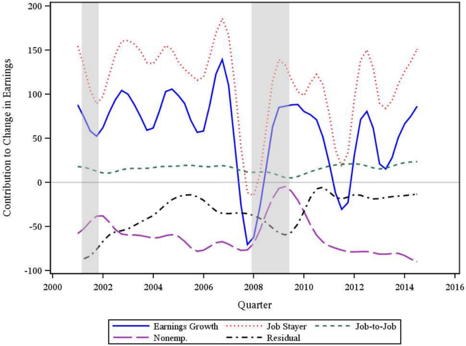
Job-to-Job Flows

- J2J big fraction of job flows (Hahn et al, 2017)



Frictional Earnings Growth

- And earnings growth (Hahn et al, 2017)



The Burdett-Mortensen OTJS Model

- Extend McCall model to include ability to search while employed.
- (We'll stick with our notation rather than using theirs)
- Basic idea:
  1. Workers can be in one of two states: employed or unemployed, with value functions V, U.
  2. Firms post wages, i.e., a given distribution of wages, $w \in [\underline{w}, \bar{w}], w \sim F(.)$.
  3. Unemployed receive job offers at exogenous rate α, no prior info.
  4. Employed receive job offers at exogenous rate λ, no prior info.
  5. Separate two ways: exogenously (rate δ) and via thru OTJS (rate $\lambda[1 - F(w)]$)
  6. Linear utility: $u(c) = b$ or $u(c) = w$.
- Important assumption: outside offers are unverifiable.
- i.e., firms cannot respond to outside offers.
- Intuitively, are there parameter values for which $w_R = \underline{w}$?

Ex Ante Homogeneous Model ("The BM Model")

- We'll see two key components to matching flows:
  1. The job-finding rate $\alpha[1 - F(w_R)]$
  2. The worker-finding rate $l(w|w_R, F)$
- Flow value of unemployment:

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V(x) - U, 0\} dF(x)$$

- Employment:

$$rV(w) = w + \lambda \int_{\underline{w}}^{\bar{w}} \max\{V(x) - V(w), 0\} dF(x) + \delta(U - V(w))$$

- Over what range will the employed individual accept a job?

Reservation Strategy

- Reservation wage: $V(w_R) = U$:

$$\begin{aligned} rV(w_R) &= w_R + \lambda \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x) \\ &\quad + \delta(U - V(w_R)) \\ rV(w_R) &= rU = w_R + \lambda \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x) \\ \Rightarrow rU - rV(w_R) &= 0 = b - w_R \\ &\quad + (\alpha - \lambda) \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x) \\ \Rightarrow w_R &= b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} (V(x) - V(w_R))dF(x) \end{aligned}$$

- If $\alpha = \lambda$, what is the interpretation?

Reservation Strategy II

- Integration by parts:

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} (V(x) - V(w_R)) dF(x)$$

$$\begin{aligned}
\int_{w_R}^{\bar{w}} (V(x) - V(w_R)) dF(x) = (V(x) - V(w_R)) F(x)|_{w_R}^{\bar{w}} \\
- \int_{w_R}^{\bar{w}} V'(x) F(x) dx \\
= (V(\bar{w}) - V(w_R)) F(\bar{w}) - \int_{w_R}^{\bar{w}} V'(x) F(x) dx \\
= \int_{w_R}^{\bar{w}} V'(x) [1 - F(x)] dx
\end{aligned}$$

$$\rightarrow w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} V'(x) [1 - F(x)] dx$$

Reservation Strategy III

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} V'(x)[1 - F(x)]dx$$

What is the derivative of the employed value function?

- $$rV(w) = w + \lambda \int_{\underline{w}}^{\bar{w}} \max\{V(x) - V(w), 0\} dF(x) + \delta(U - V(w))$$
  $$\rightarrow r\frac{\partial V}{\partial w} = dw - \lambda[1 - F(w_R)]\frac{\partial V}{\partial w} - \delta\frac{\partial V}{\partial w}$$
  $$\rightarrow \frac{\partial V}{\partial w} = \frac{dw}{r + \delta + \lambda[1 - F(w_R)]}$$
- Thus, the reservation wage is

$$w_R = b + (\alpha - \lambda) \int_{w_R}^{\bar{w}} \frac{[1 - F(x)]}{r + \delta + \lambda[1 - F(w_R)]} dx$$

Steady-States

- Flow unemployment equation:

$$\dot{u} = \delta(1-u) - \alpha[1-F(w_R)]u$$
$$u = \frac{\delta}{\delta + \alpha[1-F(w_R)]}$$

- Wage distribution?
- Key feature of this model: can characterize the steady-state wage distribution from model primitives.
- Define the CDF of employed wages at time t, $G(w,t)$ and population wage distribution $G(w,t)(1-u(t))$:

$$\frac{\partial G(w,t)(1-u(t))}{\partial t} = \alpha \max\{F(w) - F(w_R), 0\}u(t) - [\delta + \lambda(1-F(w))]G(w,t)(1-u(t))$$

- $G(w,t)$: prop. of workers receiving no greater than w.

Wage Dispersion in Steady-State

- In steady-state:

$$\frac{\partial G(w,t)(1-u(t))}{\partial t} = 0 = \alpha \max\{F(w) - F(w_R), 0\}u(t) \\ - [\delta + \lambda(1-F(w))]G(w,t)(1-u(t))$$

- For $w \geq w_R$:

$$[\delta + \lambda(1-F(w))]G(w)(1-u(t)) = \alpha[F(w) - F(w_R)]u(t)$$

$$\rightarrow G(w) = \frac{\alpha(F(w) - F(w_R))}{[\delta + \lambda(1-F(w))]}\frac{u(t)}{1-u(t)}$$

Wage Dispersion in Steady-State

- Combining unemployment and the wage distribution:

$$
\begin{aligned}
G(w) &= \frac{\alpha(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))]} \frac{u(t)}{1 - u(t)} \\
u &= \frac{\delta}{\delta + \alpha[1 - F(w_R)]} \\
\rightarrow G(w) &= \frac{\alpha(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))]} \frac{\delta}{\alpha[1 - F(w_R)]} \\
&= \frac{\delta(F(w) - F(w_R))}{[\delta + \lambda(1 - F(w))][1 - F(w_R)]}
\end{aligned}
$$

- Intuitively, what does this say?
- What does $\lambda = 0$ imply about this distribution?

The Firm

- What role does the firm play in all of this?
- How will it choose which wages to post?
- Firm profit function:

$$\pi = \max_{w} (p - w) l(w|w_R, F)$$
$$\pi(w|w_R, F) = (p - w) l(w|w_R, F)$$

- What is l? The probability that a firm will meet a worker if it posts a wage w, given
  1. the reservation wage
  2. the distribution of other firm's wages
- This is an equal profit condition (ie, like the free entry condition).

Worker Finding Rate

- What is $l(w|w_R, F)$?
- In steady-state, workers per firm earning a wage w, i.e., probability of finding a worker if you post a wage, w.
- Inflows:

$$\alpha u + (1-u)\lambda G(w,t)$$

- Outflows:

$$(\delta + \lambda[1 - F(w)])$$

- Define $m = \frac{workers}{firms}$. Then,

$$(\delta + \lambda[1 - F(w)])l(w|w_R, F) = m[\alpha u + (1-u)\lambda G(w,t)]$$

$$(\delta + \lambda[1 - F(w)])l(w|w_R, F) = m\alpha \frac{\delta}{\delta + \alpha[1 - F(w_R)]} + \frac{m\alpha[1 - F(w_R)]}{\delta + \alpha[1 - F(w_R)]}\lambda G(w,t)]$$

Worker Finding Rate II

- What is $l(w|w_R, F)$?

$$(\delta + \lambda[1 - F(w)])l(w|w_R, F) = m\alpha \frac{\delta}{\delta + \alpha[1 - F(w_R)]}$$

$$+ \frac{m\alpha[1 - F(w_R)]}{\delta + \alpha[1 - F(w_R)]}\lambda G(w)]$$

$$l(w|w_R, F) = m\alpha \frac{\delta + [1 - F(w_R)]\lambda G(w)}{(\delta + \lambda[1 - F(w)])(\delta + \alpha[1 - F(w_R)])}$$

$$l(w|w_R, F) = m\alpha \frac{\delta(1 + \lambda \frac{[F(w) - F(w_R)]}{[\delta + \lambda(1 - F(w))]})}{(\delta + \lambda[1 - F(w)])(\delta + \alpha[1 - F(w_R)])}$$

$$l(w|w_R, F) = m\alpha\delta \frac{\frac{\delta + \lambda(1 - F(w_R))}{\delta + \alpha(1 - F(w_R))}}{(\delta + \lambda[1 - F(w)])^2}$$

Equilibrium

- From the paper:
  An equilibrium solution to the search and wage-posting game outlined above can be described by a triple (w_R, F, π) such that w_R, the common reservation wage of unemployed workers is the solution to the optimal stopping-time problem out of unemployment, and π is the value function given by the optimal choice of w, and the distribution F is such that

$$\begin{aligned} (p-w)l(w|w_R,F) &= \pi \quad \text{for all w on the support of F} \\ (p-w)l(w|w_R,F) &\leq \pi \quad \text{otherwise} \end{aligned}$$

- This is essentially a "free entry" condition.

Analytical Characterizations

- Assume that $\lambda = \alpha$, i.e., OTJS as efficient as off-the-job.
- Then, $w_R = b$.

$$l(w|w_R, F) = m\alpha\delta \frac{\frac{\delta+\lambda(1-F(w_R))}{\delta+\alpha(1-F(w_R))}}{(\delta + \lambda[1 - F(w)])^2}$$

$$l(w|w_R, F) = \frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2}$$

$$\rightarrow (p - w)l(w|w_R, F) = (p - w)\frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2}$$

Analytical Characterizations

- In equilibrium, profits are all equal:
  $(p - w)l(w|w_R, F) = (p - w_R)l(w_R|w_R, F)$

$$(p - w)l(w|w_R, F) = (p - w_R)\frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2}$$

$$\rightarrow (p - w)\frac{m\alpha\delta}{(\delta + \alpha[1 - F(w)])^2} = (p - w_R)\frac{m\alpha\delta}{(\delta + \alpha[1 - \cancel{F(w_R)}])^2}$$

$$\rightarrow (\delta + \alpha[1 - F(w)])^2 = \frac{(p - w)}{p - w_R}(\delta + \alpha)^2$$

$$\rightarrow [1 - F(w)] = \frac{(\delta + \alpha)}{\alpha}[\frac{(p - w)}{p - w_R}]^{\frac{1}{2}} - \frac{\delta}{\alpha}$$

$$\rightarrow F(w) = 1 + \frac{\delta}{\alpha} - \frac{(\delta + \alpha)}{\alpha}[\frac{(p - w)}{p - w_R}]^{\frac{1}{2}}$$

$$\rightarrow F(w) = \frac{(\delta + \alpha)}{\alpha}(1 - [\frac{(p - w)}{p - w_R}]^{\frac{1}{2}})$$

- We've solved for the offered wage distribution!

Equilibrium Wage Dispersion

- Found that $G(w)$ given by

$$G(w) = \frac{\delta(F(w) - F(w_R))}{[\delta + \alpha(1 - F(w))][1 - F(w_R)]}$$

$$G(w) = \frac{\delta F(w)}{[\delta + \alpha(1 - F(w))]}$$

- And the offer distribution given by

$$F(w) = \frac{(\delta + \alpha)}{\alpha}(1 - [\frac{(p - w)}{p - w_R}]^{\frac{1}{2}})$$

- Thus, the equilibrium wage distribution is given by

$$G(w) = \frac{\delta \frac{(\delta+\alpha)}{\alpha}(1 - [\frac{(p-w)}{p-w_R}]^{\frac{1}{2}})}{[\delta + \alpha(1 - \frac{(\delta+\alpha)}{\alpha}(1 - [\frac{(p-w)}{p-w_R}]^{\frac{1}{2}}))]}$$

Next Time

- Disciplining wage contracts: risk aversion.
- Between now and then:
  1. Access the campus storage/cluster.
  2. Run some example code.
  3. Create a folder in "user_folders" called first initial-last name (i.e., fvonclownstick).
- Start your introduction/research proposal project.
- Will have presentations of these in a couple of weeks.