Quantitative Macro-Labor: Wage Dispersion and Comparative Statics

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Announcements

- Ducks won: I'll delay the homework... for now.

Recap: The McCall Model

- Basic idea:
  1. Workers can be in one of two states: employed or unemployed, with value functions V, U.
  2. Receive job offers at exogenous rate α, no information about meeting prior.
  3. Once employed, workers remain at current job until unexogenously separated (no OTJS) at rate δ.
  4. Exogenous distribution of wages, $w \in [\underline{w}, \bar{w}], w \sim F(.)$.
  5. Linear utility: $u(c) = b$ or $u(c) = w$.
- Optimal policy is a "reservation strategy," i.e., a lower bound on the wages a worker will accept out of unemployment.
- Why is $w_R > \underline{w}$?

Model and Reservation Strategy

- Generally, we will use the continuous time Bellman in its "asset value" formulation:

$$U = \frac{b + \alpha E[\max\{V, U\}]}{r + \alpha} \quad (1)$$

$$(r + \alpha)U = b + \alpha E[\max\{V, U\}] \quad (2)$$

$$rU = b + \alpha E[\max\{V - U, 0\}] \quad (3)$$

$$rU = b + \alpha \int_{\underline{w}}^{\bar{w}} \max\{V - U, 0\} dF(w) \quad (4)$$

- Employment:

$$rV(w) = w - \delta(V(w) - U) \quad (5)$$

- Reservation strategy:

$$w_R = b + \frac{\alpha}{r + \delta} \int_{w_R}^{\bar{w}} [1 - F(w)] dw \quad (6)$$

Reservation Strategy II

- Reservation strategy:

$$w_R = b + \frac{\alpha}{r+\delta}\int_{w_R}^{\bar{w}}[1 - F(w)]dw \tag{7}$$

$$w_R = b + \frac{\alpha}{r+\delta}\int_{w_R}^{\bar{w}}(w - w_R)dF(w) \tag{8}$$

- Assume that the distribution of wage offers is uniform.
- What is the conditional expectation of a truncated uniform random variable? $E[w - w_R | w \geq w_R] = \frac{\bar{w} - w_R}{2}$
- What is the probability of drawing from the truncated part of the offer distribution? $P(w \geq w_R) = \frac{\bar{w} - w_R}{\bar{w} - \underline{w}}$.

$$w_R = b + \frac{\alpha}{r+\delta}\frac{\bar{w} - w_R}{2}\frac{\bar{w} - w_R}{\bar{w} - \underline{w}} \tag{9}$$

$$w_R = b + \frac{\alpha}{r+\delta}\frac{(\bar{w} - w_R)^2}{2(\bar{w} - \underline{w})} \tag{10}$$

- (yes, this is the same as if you integrate the option value)

Reservation Strategy II

- Reservation strategy:

$$w_R = b + \frac{\alpha}{r+\delta}\frac{(\bar{w} - w_R)^2}{2(\bar{w} - \underline{w})} \tag{11}$$

$$w_R = b + \frac{\alpha}{r+\delta}\frac{\bar{w}^2 - \bar{w}w_R + w_R^2}{2(\bar{w} - \underline{w})} \tag{12}$$

$$0 = b + \frac{\alpha}{r+\delta}\frac{\bar{w}^2}{2(\bar{w} - \underline{w})} - (1 + \frac{\alpha}{r+\delta}\frac{\bar{w}}{2(\bar{w} - \underline{w})})w_R \tag{13}$$

$$+ \frac{\alpha}{r+\delta}\frac{1}{2(\bar{w} - \underline{w})}w_R^2 \tag{14}$$

- Apply quadratic formula and choose root st $w_R \in [0, 1]$

$$\frac{(1 + \frac{\alpha}{r+\delta}\frac{\bar{w}}{2(\bar{w}-\underline{w})}) \pm \sqrt{(1 + \frac{\alpha}{r+\delta}\frac{\bar{w}}{2(\bar{w}-\underline{w})})^2 - 4(b + \frac{\alpha}{r+\delta}\frac{\bar{w}^2}{2(\bar{w}-\underline{w})})(\frac{\alpha}{r+\delta}\frac{1}{2(\bar{w}-\underline{w})})}}{2(b + \frac{\alpha}{r+\delta}\frac{\bar{w}^2}{2(\bar{w}-\underline{w})})} \tag{15}$$

Reservation Strategy III

- Reservation strategy:

$$\frac{(1+\frac{\alpha}{r+\delta}\frac{\bar{w}}{2(\bar{w}-\underline{w})}) \pm \sqrt{(1+\frac{\alpha}{r+\delta}\frac{\bar{w}}{2(\bar{w}-\underline{w})})^2 - 4(b+\frac{\alpha}{r+\delta}\frac{\bar{w}^2}{2(\bar{w}-\underline{w})})(\frac{\alpha}{r+\delta}\frac{1}{2(\bar{w}-\underline{w})})}}{2(b+\frac{\alpha}{r+\delta}\frac{\bar{w}^2}{2(\bar{w}-\underline{w})})} \tag{16}$$

- Let's just pick some values for the parameters (assume monthly calibration):
  1. $w \sim U[0,1]$
  2. $\alpha = 0.43$: avg. mon. U-E (this isn't right. Why?)
  3. $\delta = 0.03$: avg. mon. E-U
  4. $r = 0.0041$: ann. int. rate
  5. $b = 0.4$: UI rep. rate
- $w_R \in \{1.31, 0.72\}$
- I'm a little skeptical of these results, but you get the idea.

Hazard Rate

- What is the hazard rate of unemployment?
- Rate of leaving unemployment at time t.

$$
\begin{aligned}
H_u(t) &= \alpha \int_{w_R}^{\bar{W}} dF(w) && (17)\\
&= \alpha(F(\bar{w}) - F(w_R)) && (18)\\
&= \underbrace{\alpha}_{MeetingRate} \underbrace{(1 - F(w_R))}_{Selectivity} && (19)
\end{aligned}
$$

- Note, almost every search model generates a hazard composed of the product of a meeting probability and worker selectivity.
- Hazard rate of employment (leaving employment for unemployment)?

$$H_e(t) = \delta \tag{20}$$

- Because separations are independent of state.

Dynamics of Unemployment

- Use hazard rates to understand dynamics and steady-state.
- What does the model predict about employment and unemployment?

$$\dot{u} = \delta(1-u) - \alpha(1-F(w_R))u \tag{21}$$

$$\dot{e} = \alpha(1-F(w_R))(1-e) - \delta e \tag{22}$$

- Steady-state: $\dot{u} = 0$, $\dot{e} = 0$:

$$0 = \delta(1-u) - \alpha(1-F(w_R))u \tag{23}$$

$$\rightarrow u = \frac{\delta}{\delta + \alpha(1-F(w_R))} \tag{24}$$

$$0 = \alpha(1-F(w_R))(1-e) - \delta e \tag{25}$$

$$\rightarrow e = \frac{\alpha(1-F(w_R))}{\alpha(1-F(w_R)) + \delta} \tag{26}$$

What can we say about an increase in UI?

- Whenever we write down a model, we have created a laboratory.
- Let's run experiments with it!
- What will happen to wages and unemployment if UI b increases?
- For wages, all we need to know is the change in the reservation strategy:

$$\frac{\partial w_R}{\partial b} = 1 + \frac{\alpha}{r+\delta} \frac{\partial \int_{w_R}^{\bar{w}} [1 - F(w)] dw}{\partial w_R} \frac{\partial w_R}{\partial b} \tag{27}$$

- Leibniz's integral rule:

$$\frac{\partial}{\partial x} \int_{a(x)}^{b(x)} f(x,t)dt = f(x, b(x)) \frac{\partial b(x)}{\partial x} - f(x, a(x)) \frac{\partial a(x)}{\partial x} \tag{28}$$

$$+ \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,t)dt \tag{29}$$

$\frac{\partial w_R}{\partial b}$

- For wages, all we need to know is the change in the reservation strategy:

$$\frac{\partial w_R}{\partial b} = 1 + \frac{\alpha}{r+\delta} \frac{\partial \int_{w_R}^{\bar{w}} [1 - F(w)] dw}{\partial w_R} \frac{\partial w_R}{\partial b} \tag{30}$$

- Leibniz's integral rule:

$$\frac{\partial}{\partial x} \int_{a(x)}^{b(x)} f(x,t) dt = f(x, b(x)) \frac{\partial b(x)}{\partial x} - f(x, a(x)) \frac{\partial a(x)}{\partial x} + \int_{a(x)}^{b(x)} \frac{\partial f(x,t)}{\partial x} dt$$

$$\begin{aligned} \frac{\partial w_R}{\partial b} &= 1 + \frac{\alpha}{r+\delta} \Big(\cancel{[1 - F(\bar{w})] \frac{\partial \bar{w}}{\partial w_R}} \\ &\quad - [1 - F(w_R)] \frac{\partial w_R}{\partial w_R} + \cancel{\int_{w_R}^{\bar{w}} \frac{\partial [1 - F(w)]}{\partial w_R}} \Big) \\ \frac{\partial w_R}{\partial b} &= 1 - \frac{\alpha}{r+\delta} [1 - F(w_R)] \frac{\partial w_R}{\partial b} \\ \frac{\partial w_R}{\partial b} &= \frac{r+\delta}{r+\delta+\alpha[1 - F(w_R)]} < 1 \end{aligned} \tag{31}$$

What about unemployment?

- Now, $\frac{\partial u}{\partial b}$.
- Let's find the semi-elasticity: $\frac{\partial log(u)}{\partial b}$

$$ln(u) = ln(\delta) - ln(\delta + \alpha(1 - F(w_R))) \tag{32}$$

$$\frac{\partial ln(u)}{\partial b} = \frac{\alpha f(w_R)\frac{\partial w_R}{\partial b}}{\delta + \alpha(1 - F(w_R))} \tag{33}$$

- Unemployment clearly increases.
- More interesting: separation rate (δ) and offer arrival rate (α)
- Why? Predictions are unclear.
- If $\alpha \uparrow$, find jobs faster, but also sample better jobs more often.

$\frac{\partial w_R}{\partial \alpha}$

- For wages, all we need to know is the change in the reservation strategy:

$$\frac{\partial w_R}{\partial \alpha} = \underbrace{\frac{1}{r+\delta}\int_{w_R}^{\bar{w}}[1-F(w)]dw}_{Match\ Rate} - \underbrace{\frac{\alpha}{r+\delta}[1-F(w_r)]\frac{\partial w_R}{\partial \alpha}}_{Selectivity} \tag{34}$$

$$\frac{\partial w_R}{\partial \alpha} = \frac{\int_{w_R}^{\bar{w}}[1-F(w)]dw}{r+\delta+\alpha[1-F(w_r)]} \tag{35}$$

- Now the semi-elasticity: $\frac{\partial log(u)}{\partial \alpha}$

$$ln(u) = ln(\delta) - ln(\delta + \alpha(1-F(w_R))) \tag{36}$$

$$\frac{\partial ln(u)}{\partial \alpha} = \underbrace{\frac{\alpha f(w_R)\frac{\partial w_R}{\partial \alpha}}{\delta+\alpha(1-F(w_R))}}_{Selectivity} - \underbrace{\frac{(1-F(w_R))}{\delta+\alpha(1-F(w_R))}}_{Match\ Rate} \tag{37}$$

Log-Concavity

$$\frac{\partial ln(u)}{\partial \alpha} = \frac{\alpha f(w_R) \frac{\int_{w_R}^{\bar{w}} [1-F(w)]dw}{r+\delta+\alpha[1-F(w_r)]}}{\delta + \alpha(1 - F(w_R))} - \frac{(1 - F(w_R))}{\delta + \alpha(1 - F(w_R))} \quad (38)$$

- Uh oh... how are we going to sign this?
- Properties of log-concave distributions (where $F(x)$ is log-concave):
    1. $F(x)$ log-concave $\rightarrow \int F(x)$ log-concave.
    2. $F(x)$ log-concave $\rightarrow \frac{\partial F}{\partial x}$ log-concave.
    3. $F(x)F''(x) \leq (F'(x))^2$

Log-Concavity

- Properties of log-concave distributions (where $F(x)$ is log-concave):
  1. $F(x)$ log-concave $\rightarrow \int F(x)$ log-concave.
  2. $F(x)$ log-concave $\rightarrow \frac{\partial F}{\partial x}$ log-concave.
  3. $F(x)F''(x) \leq (F'(x))^2$
- $(\delta + \alpha(1 - F(w_R))) > 0$

$$\frac{\partial ln(u)}{\partial \alpha} \propto \alpha f(w_R) \frac{\int_{w_R}^{\bar{w}} [1 - F(w)] dw}{r + \delta + \alpha[1 - F(w_r)]} - (1 - F(w_R)) \quad (39)$$

$$\propto \alpha f(w_R) \int_{w_R}^{\bar{w}} [1 - F(w)] dw - (1 - F(w_R))^2 < 0 \quad (40)$$

- By the third property of log-concave distributions.

“Estimation”/Calibration

- Earlier, I picked some parameters from Hornstein, Krusell, and Violante (“Calibrated Example”)
- If we want to match this model to the data, what targets can we use?
- Unconditional moments (i.e., population averages):
  - Hazard rates (U-E, U-E)
  - Employment rates (e, u)
  - Wage distribution
- How many moments do we need?
  - δ: separation rate
  - α: match rate
  - $F(w)$: distribution function
- Can we separately (ex-ante) identify them?
- Particularly, what can we use to identify α and $F(w)$?

“Estimation”/Calibration II

- What targets can we use to discipline model?
- Unconditional moments (i.e., population averages):
  - Hazard rates (U2E, E2U)
  - Employment rates (e, u)
  - Wage distribution
- What should we match?
- Depends on what we are after:
  - Transition rates: don’t target the transition rates.
  - Wage distribution: don’t target the wage distribution.
- What time period should we use?
- Steady-state: time-independent .
- We could pick any time interval and get same steady-state.
- But, pick monthly.
- Return to calibration momentarily.

Why are Similar Workers Paid Differently?

- Posed by Dale Mortensen in his book "Wage Dispersion"
- Abowd, Kramarz, and Margolis (1999): "That... observably equivalent individuals earn markedly different compensation and have markedly different employment histories–is one of the enduring features of empirical analyses of labor markets..."
- What are some possible reasons?
  1. Ability
  2. Selectivity
- What does the McCall model say is the source of wage dispersion?

A Notion of Wage Dispersion

- Extremely clever paper: Hornstein, Krusell, Violante (2011).
- Basic idea: use the mean-min (Mm) ratio for wage dispersion.
- Almost every search model has expression for the Mm ratio.
- Compare model Mm with data Mm.
- Reservation strategy:

$$w_R = b + \frac{\alpha}{r+\delta}\int_{w_R}^{\bar{w}}(w - w_R)dF(w) \tag{41}$$

$$w_R = b + \frac{\alpha(1-F(w_R))}{r+\delta}\int_{w_R}^{\bar{w}}(w - w_R)\frac{dF(w)}{1-F(w_R)} \tag{42}$$

$$w_R = b + \frac{\alpha(1-F(w_R))}{r+\delta}\int_{w_R}^{\bar{w}}(w - w_R)\frac{dF(w)}{1-F(w_R)} \tag{43}$$

- Exp. of a truncated random variable: $E[w|w \geq w_R] = \hat{w}$

$$\rightarrow w_R = b + \frac{\alpha(1-F(w_R))}{r+\delta}[\hat{w} - w_R] \tag{44}$$

The Mean-Min Ratio

- Average UI in the economy: $b = \rho \hat{w}$
- Reservation strategy:

$$w_R = \rho \hat{w} + \frac{\alpha(1 - F(w_R))}{r + \delta}[\hat{w} - w_R] \tag{45}$$

- What is minimum wage in this economy? w_R of course!

$$(\rho + \frac{\alpha(1 - F(w_R))}{r + \delta})\hat{w} = (1 + \frac{\alpha(1 - F(w_R))}{r + \delta})w_R \tag{46}$$

$$\rightarrow \frac{\hat{w}}{w_R} = \frac{1 + \frac{\alpha(1-F(w_R))}{r+\delta}}{\rho + \frac{\alpha(1-F(w_R))}{r+\delta}} \tag{47}$$

- Good news: $\rho < 1 \rightarrow$ mean wage is greater than w_R.
- What is *incredibly* useful (empirically) about this formulation?

Calibration

$$\frac{\hat{w}}{w_R} = \frac{1 + \frac{\alpha(1-F(w_R))}{r+\delta}}{\rho + \frac{\alpha(1-F(w_R))}{r+\delta}} \tag{48}$$

- It's hard to separately identify the *offer* distribution and the *accepted* offer or wage distribution.
- This expression ignores the distinction: $\alpha(1 - F(w_R)) = H_u$.
- We just need the observed hazard, and can plug in for values.

Calibration II

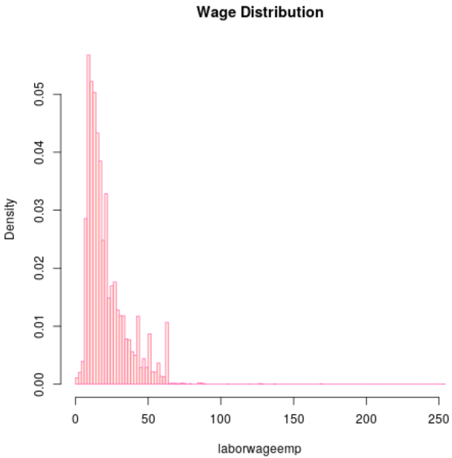
- Parameters can be calibrated directly from observed data (or observed from HKV):
  1. $\alpha(1 - F(w_R)) = 0.43$: avg. mon. U-E (HKV)
  2. $\delta = 0.03$: avg. mon. E-U (HKV)
  3. $r = 0.0041$: ann. int. rate (HKV)
  4. $\rho = 0.4$: UI rep. rate (HKV)

$$\frac{\hat{w}}{w_R} = \frac{1 + \frac{\alpha(1-F(w_R))}{r+\delta}}{\rho + \frac{\alpha(1-F(w_R))}{r+\delta}} = \frac{1 + \frac{0.43}{0.0341}}{0.4 + \frac{0.43}{0.0341}} = 1.046 \tag{49}$$

- Great! What does this tell us?
- The McCall model predicts Mm wage dispersion of 4.6%.

Wage Dispersion

- The McCall model predicts Mm wage dispersion of 4.6%.



- HKV: Mm ratio is roughly 2.
- What does this mean?

Next Time

- Extensions of the McCall model: On-the-Job Search.
- Between now and then:
  1. Access the campus storage/cluster.
  2. Run some example code.
  3. Start research proposal.